

Inverse Problems, Machine Learning and Digital Twins

Theory & Examples

Contents

- What is a Digital Twin? (Mathematically speaking)
- Digital Twins and Inverse Problems.
- Bayesian and Statistical Inversion.
- Scientific Machine Learning.
- Examples.

Digital Twin

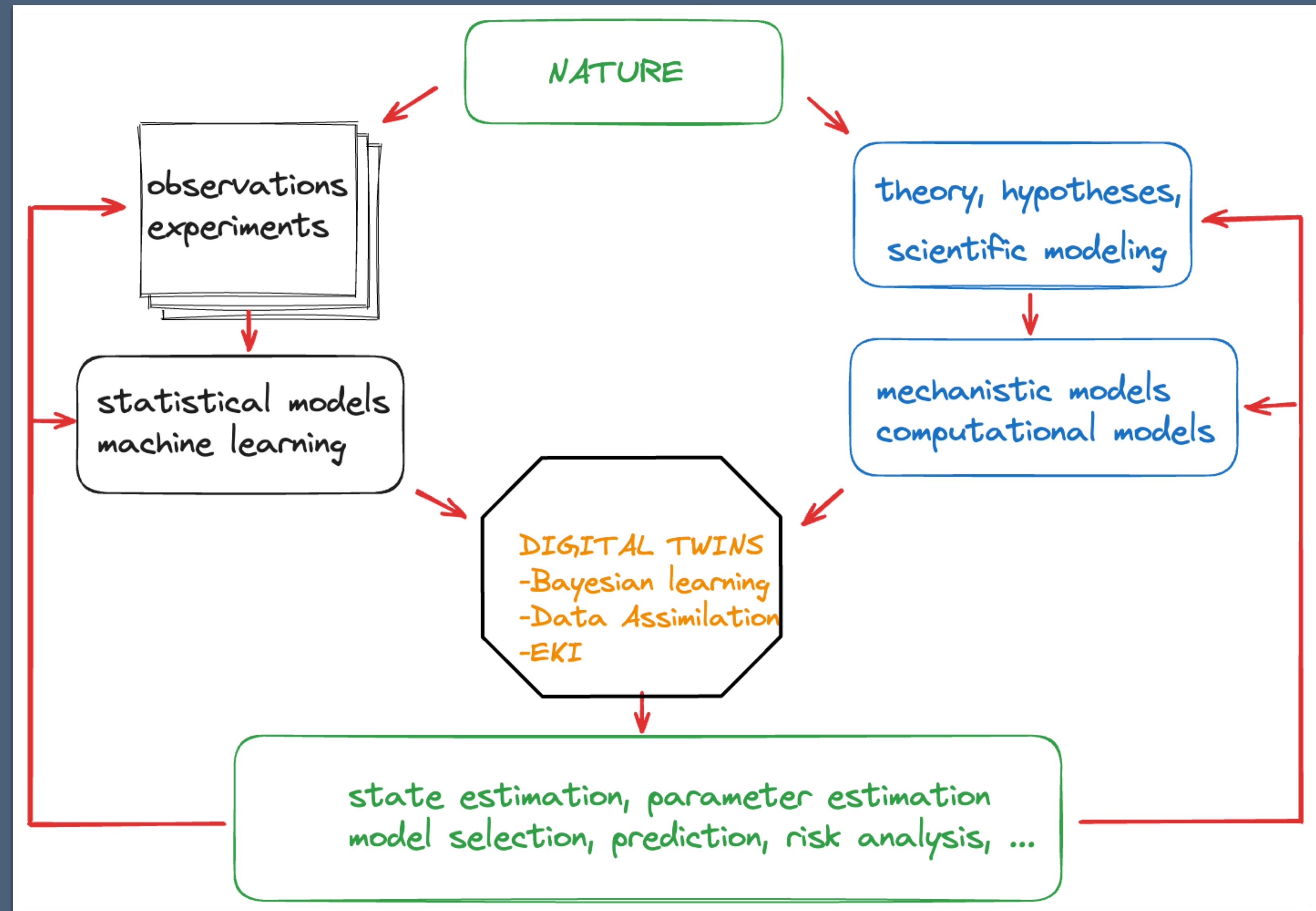
A mathematical
definition

- “A digital twin couples **computational** models with a **physical** counterpart to create a system that is dynamically updated through bidirectional **data** flows as conditions change.”
[NASEM]

Digital Twin

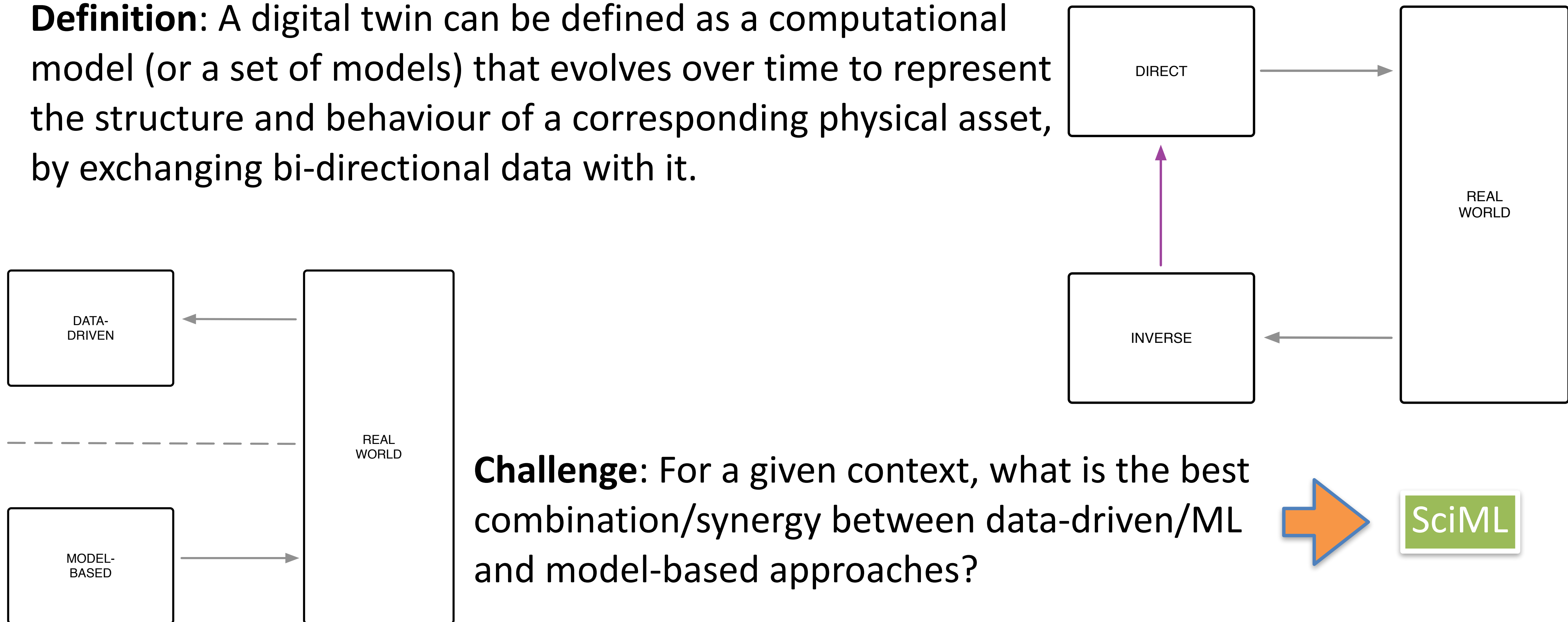
A mathematical
definition

- “A digital twin couples **computational** models with a **physical** counterpart to create a system that is dynamically updated through bidirectional **data** flows as conditions change.”
[NASEM]



Digital Twin: definition

Definition: A digital twin can be defined as a computational model (or a set of models) that evolves over time to represent the structure and behaviour of a corresponding physical asset, by exchanging bi-directional data with it.



Mathematics

Inverse Problems for DTs

Dynamical System

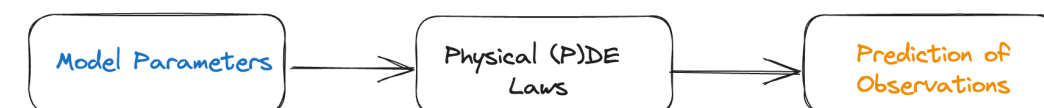
$$\frac{d\mathbf{u}}{dt} = g(t, \mathbf{u}; \boldsymbol{\theta}), \quad \mathbf{u}(t_0) = \mathbf{u}_0,$$

with g known, $\boldsymbol{\theta} \in \Theta$, $\mathbf{u}(t) \in \mathbb{R}^k$.

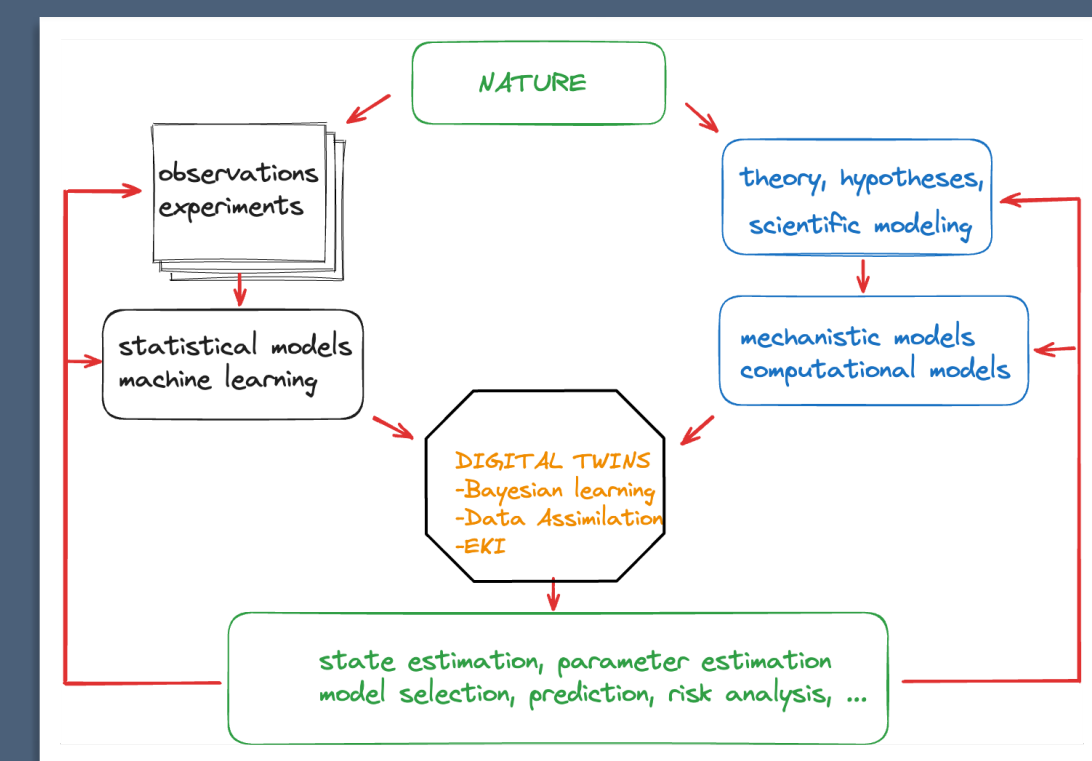
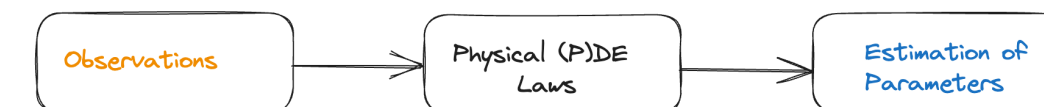
Direct: Given $\boldsymbol{\theta}$, $\mathbf{u}_0$, find $\mathbf{u}(t)$ for $t \geq t_0$.

Inverse: Given $\mathbf{u}(t)$ for $t \geq t_0$, find $\boldsymbol{\theta} \in \Theta$.

Direct Problem:



Inverse Problem:



Mathematics

Inverse Problems for DTs

Dynamical System

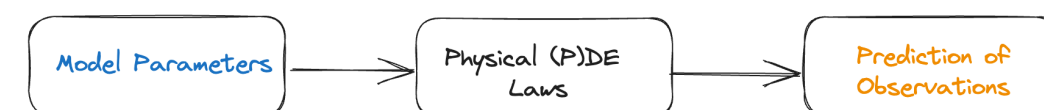
$$\frac{d\mathbf{u}}{dt} = g(t, \mathbf{u}; \boldsymbol{\theta}), \quad \mathbf{u}(t_0) = \mathbf{u}_0,$$

with g known, $\boldsymbol{\theta} \in \Theta$, $\mathbf{u}(t) \in \mathbb{R}^k$.

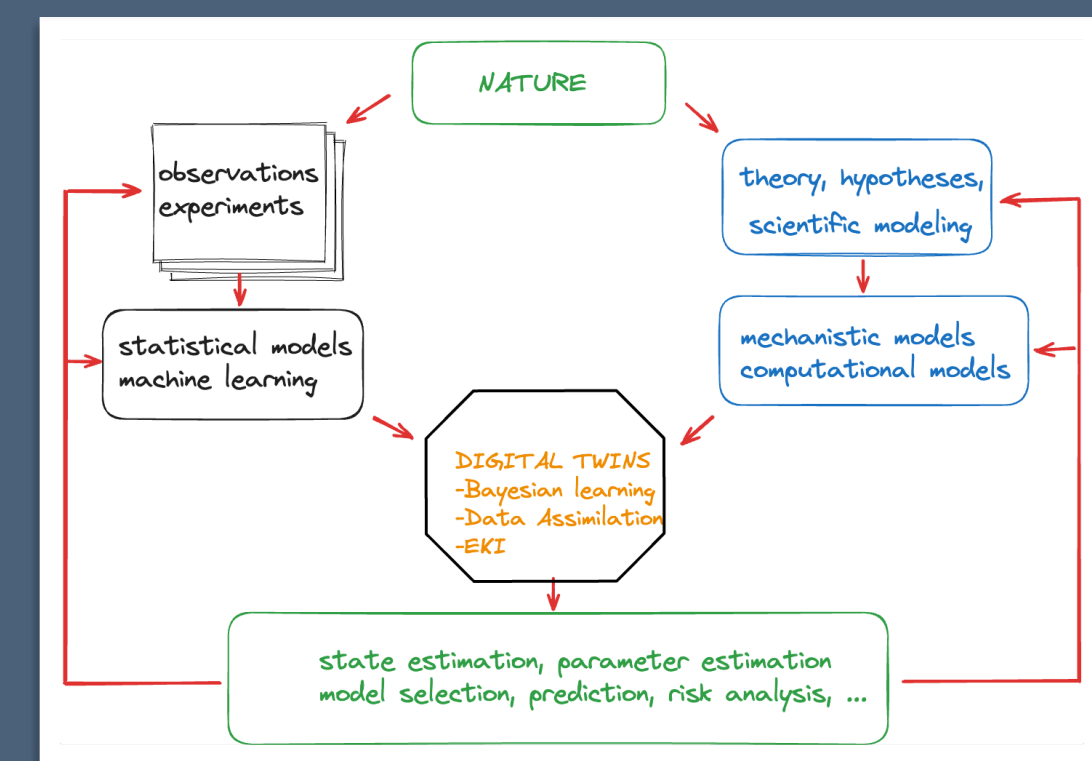
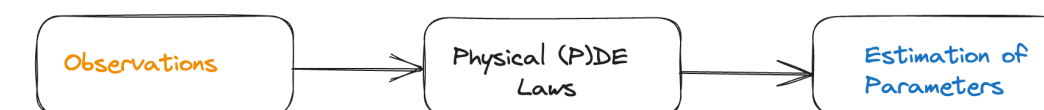
Direct: Given $\boldsymbol{\theta}$, $\mathbf{u}_0$, find $\mathbf{u}(t)$ for $t \geq t_0$.

Inverse: Given $\mathbf{u}(t)$ for $t \geq t_0$, find $\boldsymbol{\theta} \in \Theta$.

Direct Problem:



Inverse Problem:



Inverse Problems - Deterministic Case

In the deterministic case ($\eta = 0$), because of the ill-posedness of the inverse problem, we replace it by the **least-squares** optimization problem,

$$\operatorname{argmin}_{u \in X} \frac{1}{2} \|y - \mathcal{G}(u)\|_Y^2$$

that is usually **regularized** as

$$\operatorname{argmin}_{u \in E} \frac{1}{2} \left(\|y - \mathcal{G}(u)\|_Y^2 + \frac{1}{2} \|u - m_0\|_E^2 \right)$$

for a given reference point $m_0 \in E$, with E, X, Y Banach spaces. The optimization requires a **gradient** (or adjoint).

Mathematics

Inverse Problems for DTs

Dynamical System

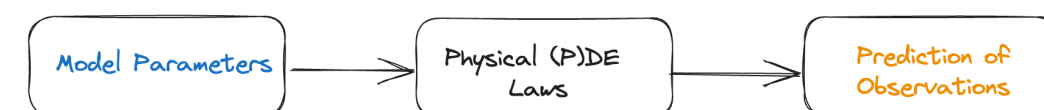
$$\frac{d\mathbf{u}}{dt} = g(t, \mathbf{u}; \boldsymbol{\theta}), \quad \mathbf{u}(t_0) = \mathbf{u}_0,$$

with g known, $\boldsymbol{\theta} \in \Theta$, $\mathbf{u}(t) \in \mathbb{R}^k$.

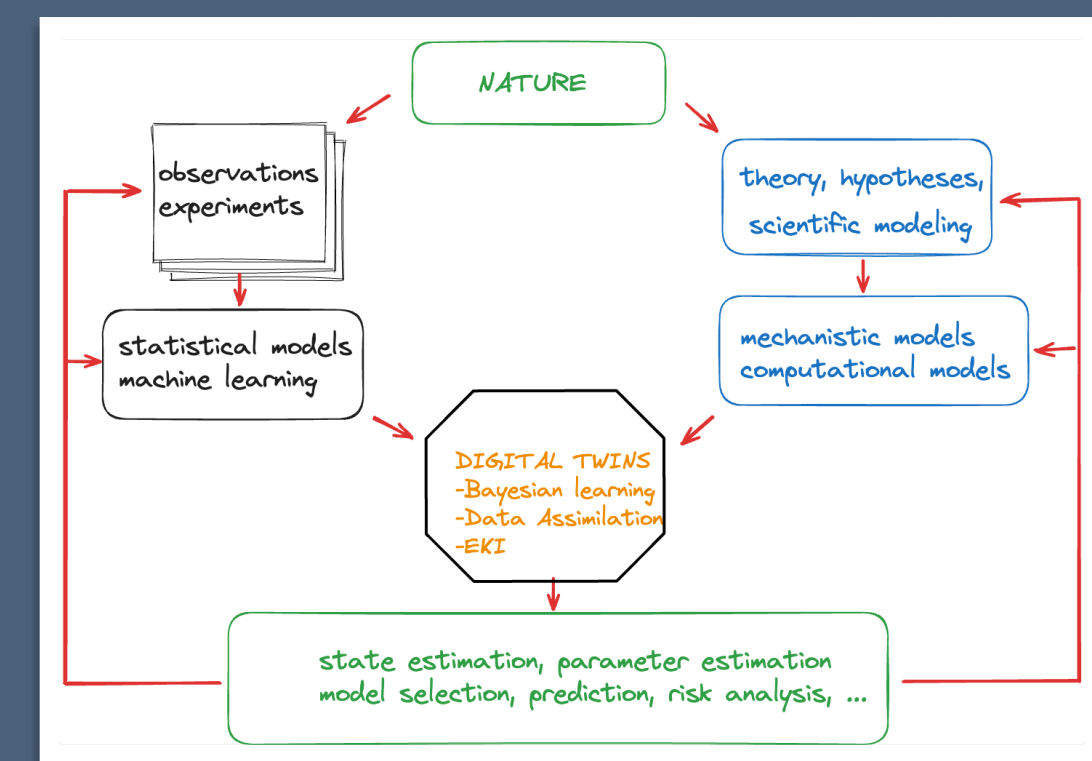
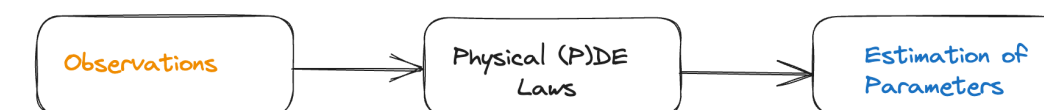
Direct: Given $\boldsymbol{\theta}$, $\mathbf{u}_0$, find $\mathbf{u}(t)$ for $t \geq t_0$.

Inverse: Given $\mathbf{u}(t)$ for $t \geq t_0$, find $\boldsymbol{\theta} \in \Theta$.

Direct Problem:



Inverse Problem:



Inverse Problems - Deterministic Case

In the deterministic case ($\eta = 0$), because of the ill-posedness of the inverse problem, we replace it by the **least-squares** optimization problem,

$$\operatorname{argmin}_{u \in X} \frac{1}{2} \|y - \mathcal{G}(u)\|_Y^2$$

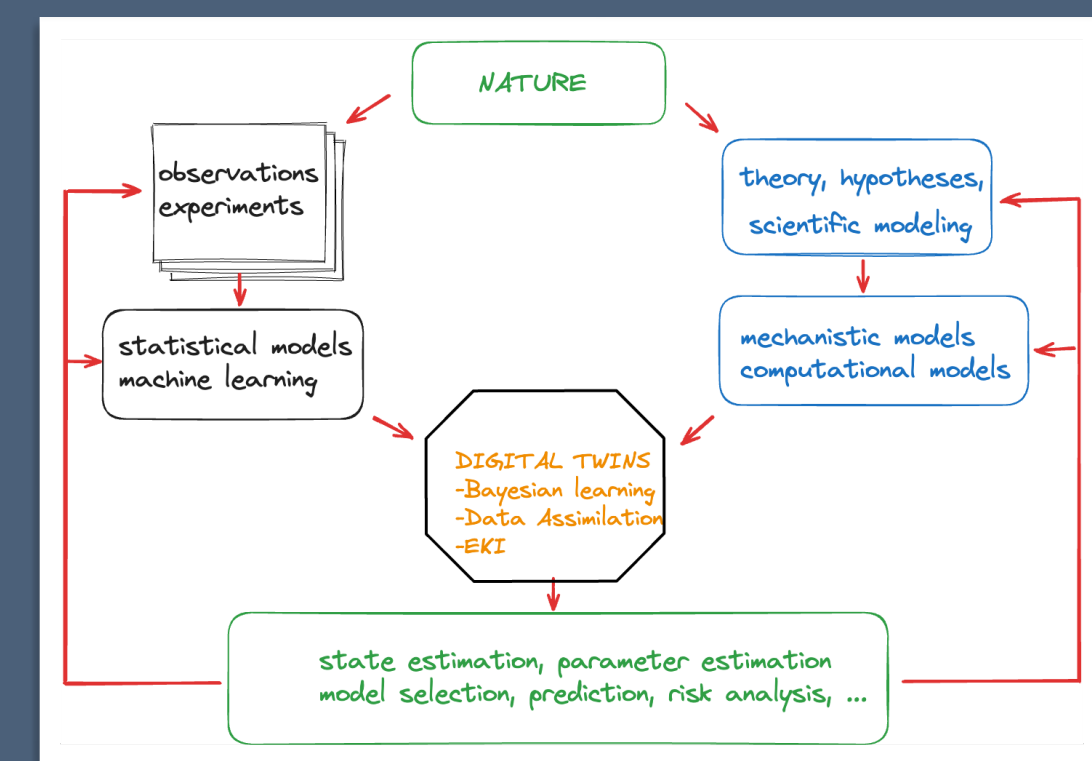
that is usually **regularized** as

$$\operatorname{argmin}_{u \in E} \frac{1}{2} \left(\|y - \mathcal{G}(u)\|_Y^2 + \frac{1}{2} \|u - m_0\|_E^2 \right)$$

for a given reference point $m_0 \in E$, with E, X, Y Banach spaces. The optimization requires a **gradient** (or adjoint).

Mathematics

Inverse Problems for DTs



Dynamical System

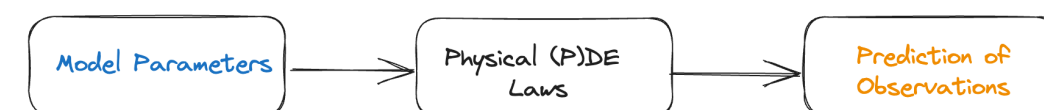
$$\frac{d\mathbf{u}}{dt} = g(t, \mathbf{u}; \boldsymbol{\theta}), \quad \mathbf{u}(t_0) = \mathbf{u}_0,$$

with g known, $\boldsymbol{\theta} \in \Theta$, $\mathbf{u}(t) \in \mathbb{R}^k$.

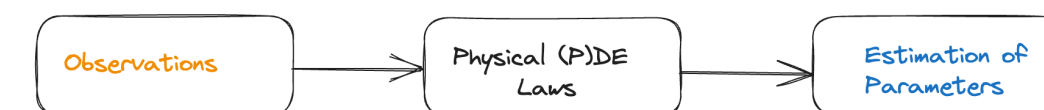
Direct: Given $\boldsymbol{\theta}$, $\mathbf{u}_0$, find $\mathbf{u}(t)$ for $t \geq t_0$.

Inverse: Given $\mathbf{u}(t)$ for $t \geq t_0$, find $\boldsymbol{\theta} \in \Theta$.

Direct Problem:



Inverse Problem:



In reality, we have **uncertainty** (noise)

- in the model,
- in the parameters,
- in the observations.

The dynamical system becomes

$$\mathbf{y} = \mathcal{G}(\mathbf{u}) + \eta,$$

where $\eta \sim \mathcal{N}(0, \Sigma)$.

Inverse Problems - Deterministic Case

In the deterministic case ($\eta = 0$), because of the ill-posedness of the inverse problem, we replace it by the **least-squares** optimization problem,

$$\operatorname{argmin}_{u \in X} \frac{1}{2} \|y - \mathcal{G}(u)\|_Y^2$$

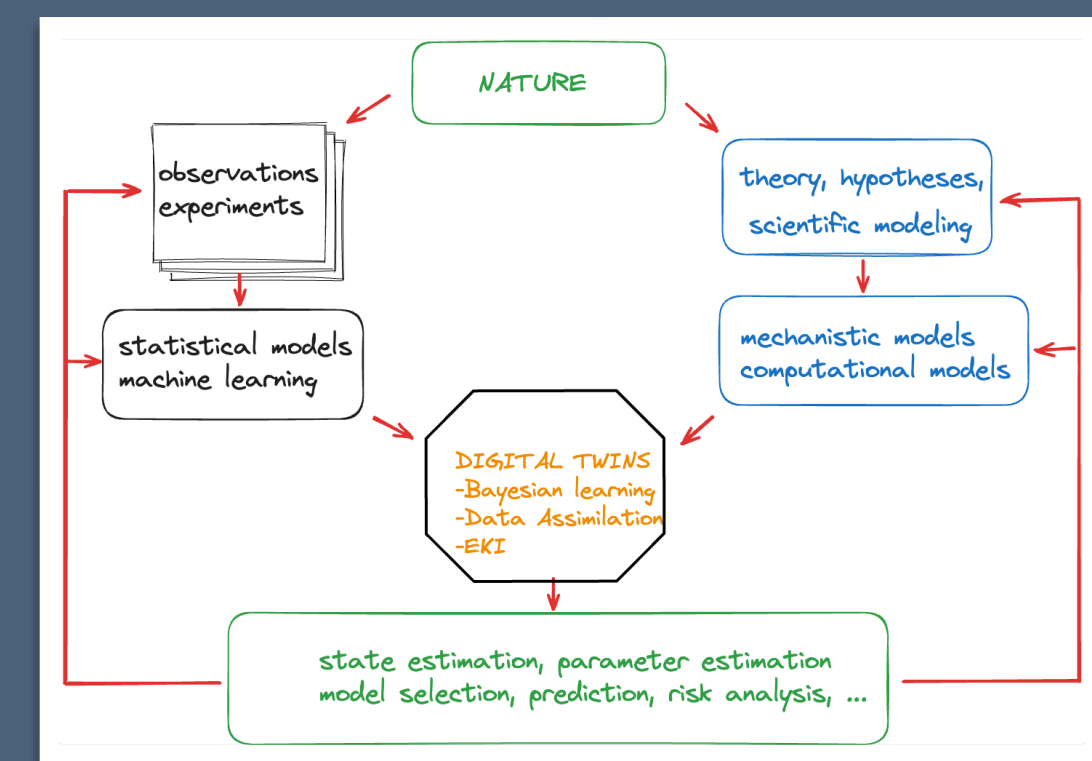
that is usually **regularized** as

$$\operatorname{argmin}_{u \in E} \frac{1}{2} \left(\|y - \mathcal{G}(u)\|_Y^2 + \frac{1}{2} \|u - m_0\|_E^2 \right)$$

for a given reference point $m_0 \in E$, with E, X, Y Banach spaces. The optimization requires a **gradient** (or adjoint).

Mathematics

Inverse Problems for DTs



Dynamical System

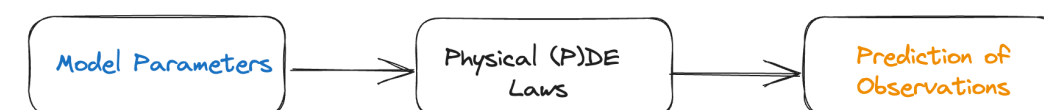
$$\frac{d\mathbf{u}}{dt} = g(t, \mathbf{u}; \boldsymbol{\theta}), \quad \mathbf{u}(t_0) = \mathbf{u}_0,$$

with g known, $\boldsymbol{\theta} \in \Theta$, $\mathbf{u}(t) \in \mathbb{R}^k$.

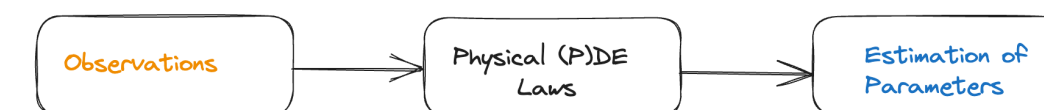
Direct: Given $\boldsymbol{\theta}$, $\mathbf{u}_0$, find $\mathbf{u}(t)$ for $t \geq t_0$.

Inverse: Given $\mathbf{u}(t)$ for $t \geq t_0$, find $\boldsymbol{\theta} \in \Theta$.

Direct Problem:



Inverse Problem:



In reality, we have **uncertainty** (noise)

- in the model,
- in the parameters,
- in the observations.

The dynamical system becomes

$$\mathbf{y} = \mathcal{G}(\mathbf{u}) + \eta,$$

where $\eta \sim \mathcal{N}(0, \Sigma)$.

Inverse Problems - Deterministic Case

In the deterministic case ($\eta = 0$), because of the ill-posedness of the inverse problem, we replace it by the **least-squares** optimization problem,

$$\operatorname{argmin}_{u \in X} \frac{1}{2} \|y - \mathcal{G}(u)\|_Y^2$$

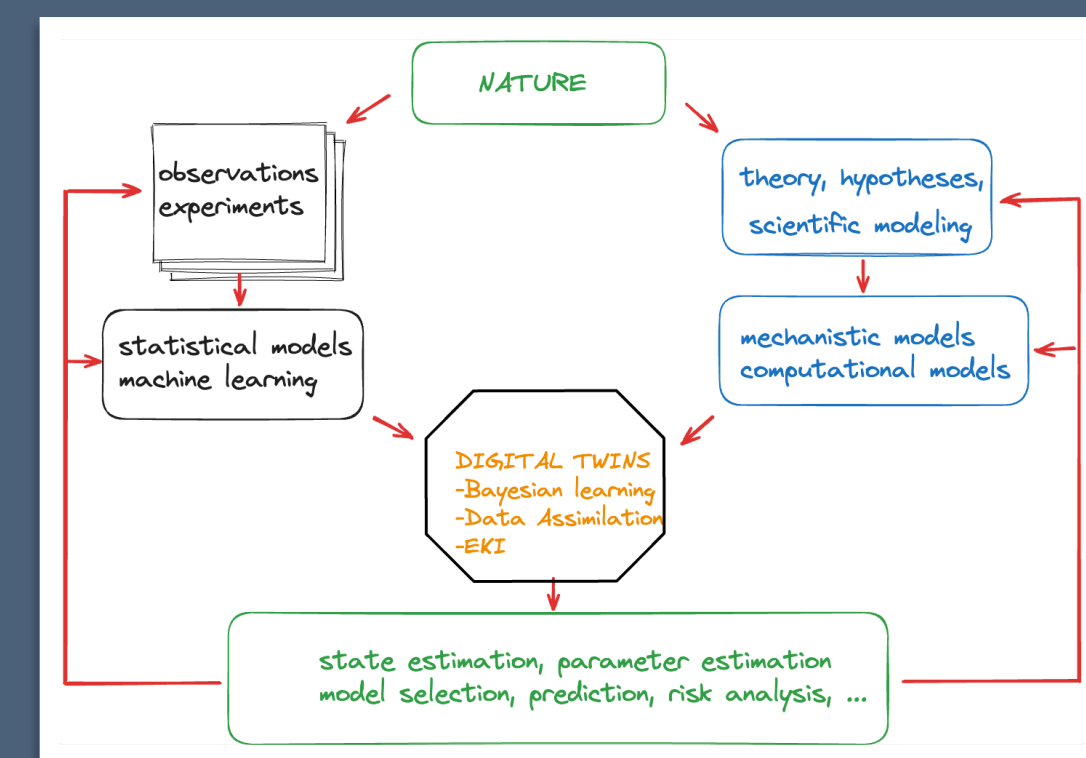
that is usually **regularized** as

$$\operatorname{argmin}_{u \in E} \frac{1}{2} \left(\|y - \mathcal{G}(u)\|_Y^2 + \frac{1}{2} \|u - m_0\|_E^2 \right)$$

for a given reference point $m_0 \in E$, with E, X, Y Banach spaces. The optimization requires a **gradient** (or adjoint).

Mathematics

Inverse Problems for DTs



Dynamical System

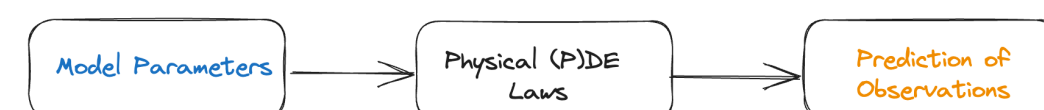
$$\frac{d\mathbf{u}}{dt} = g(t, \mathbf{u}; \boldsymbol{\theta}), \quad \mathbf{u}(t_0) = \mathbf{u}_0,$$

with g known, $\boldsymbol{\theta} \in \Theta$, $\mathbf{u}(t) \in \mathbb{R}^k$.

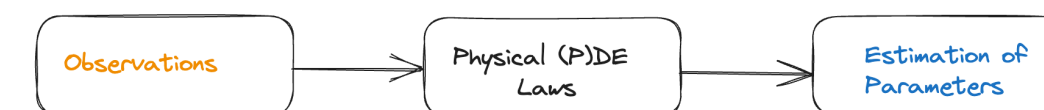
Direct: Given $\boldsymbol{\theta}$, $\mathbf{u}_0$, find $\mathbf{u}(t)$ for $t \geq t_0$.

Inverse: Given $\mathbf{u}(t)$ for $t \geq t_0$, find $\boldsymbol{\theta} \in \Theta$.

Direct Problem:



Inverse Problem:



In reality, we have **uncertainty** (noise)

- in the model,
- in the parameters,
- in the observations.

The dynamical system becomes

$$\mathbf{y} = \mathcal{G}(\mathbf{u}) + \eta,$$

where $\eta \sim \mathcal{N}(0, \Sigma)$.



Inverse Problems - Deterministic Case

In the deterministic case ($\eta = 0$), because of the ill-posedness of the inverse problem, we replace it by the **least-squares** optimization problem,

$$\operatorname{argmin}_{u \in X} \frac{1}{2} \|y - \mathcal{G}(u)\|_Y^2$$

that is usually **regularized** as

$$\operatorname{argmin}_{u \in E} \frac{1}{2} \left(\|y - \mathcal{G}(u)\|_Y^2 + \frac{1}{2} \|u - m_0\|_E^2 \right)$$

for a given reference point $m_0 \in E$, with E, X, Y Banach spaces. The optimization requires a **gradient** (or adjoint).

Inverse Problems - Stochastic Case

In the stochastic case, the solution of the inverse problem, “find u from y ,” is a **posterior probability density function** (ppdf).

Theorem (Bayes)

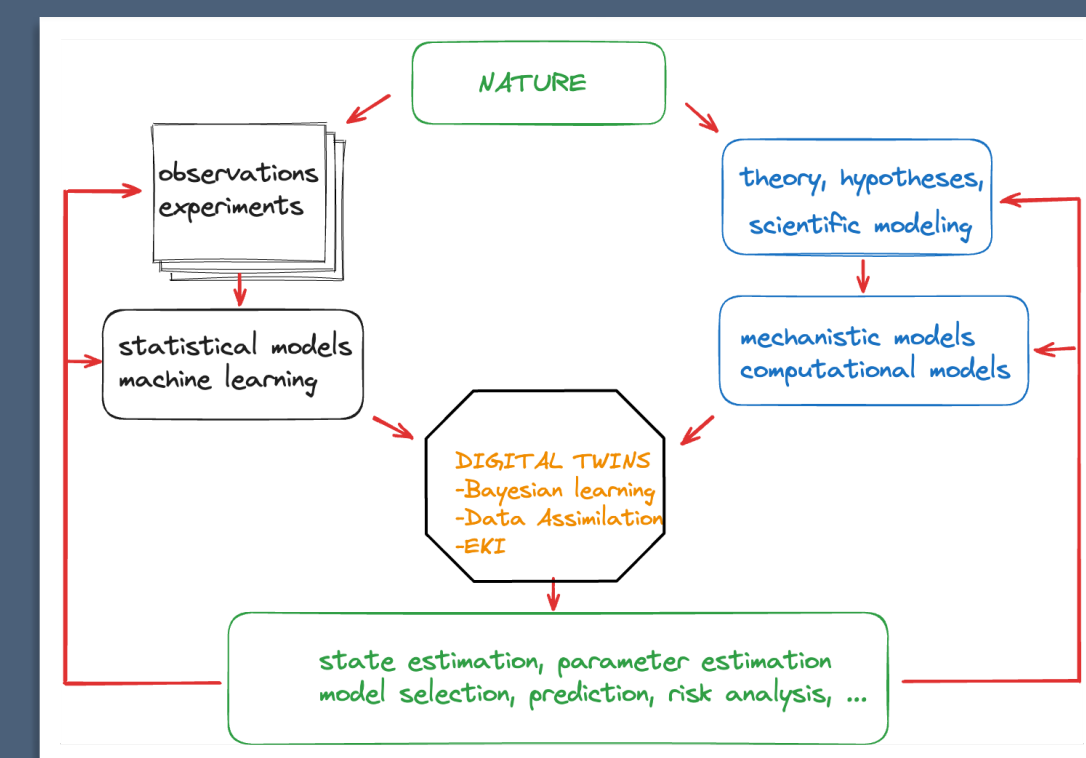
$$p(u|y) = \frac{p(y|u)p(u)}{p(y)},$$

or

$$p(\text{parameter}|\text{data}) \propto p(\text{data}|\text{parameter})p(\text{parameter}).$$

Mathematics

Inverse Problems for DTs



Dynamical System

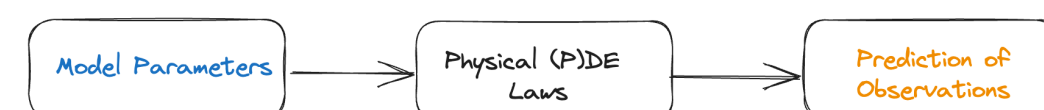
$$\frac{d\mathbf{u}}{dt} = g(t, \mathbf{u}; \boldsymbol{\theta}), \quad \mathbf{u}(t_0) = \mathbf{u}_0,$$

with g known, $\boldsymbol{\theta} \in \Theta$, $\mathbf{u}(t) \in \mathbb{R}^k$.

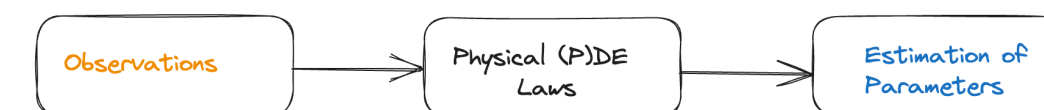
Direct: Given $\boldsymbol{\theta}$, $\mathbf{u}_0$, find $\mathbf{u}(t)$ for $t \geq t_0$.

Inverse: Given $\mathbf{u}(t)$ for $t \geq t_0$, find $\boldsymbol{\theta} \in \Theta$.

Direct Problem:



Inverse Problem:



In reality, we have **uncertainty** (noise)

- in the model,
- in the parameters,
- in the observations.

The dynamical system becomes

$$\mathbf{y} = \mathcal{G}(\mathbf{u}) + \eta,$$

where $\eta \sim \mathcal{N}(0, \Sigma)$.



Inverse Problems - Deterministic Case

In the deterministic case ($\eta = 0$), because of the ill-posedness of the inverse problem, we replace it by the **least-squares** optimization problem,

$$\operatorname{argmin}_{u \in X} \frac{1}{2} \|y - \mathcal{G}(u)\|_Y^2$$

that is usually **regularized** as

$$\operatorname{argmin}_{u \in E} \frac{1}{2} \left(\|y - \mathcal{G}(u)\|_Y^2 + \frac{1}{2} \|u - m_0\|_E^2 \right)$$

for a given reference point $m_0 \in E$, with E, X, Y Banach spaces. The optimization requires a **gradient** (or adjoint).

Inverse Problems - Stochastic Case

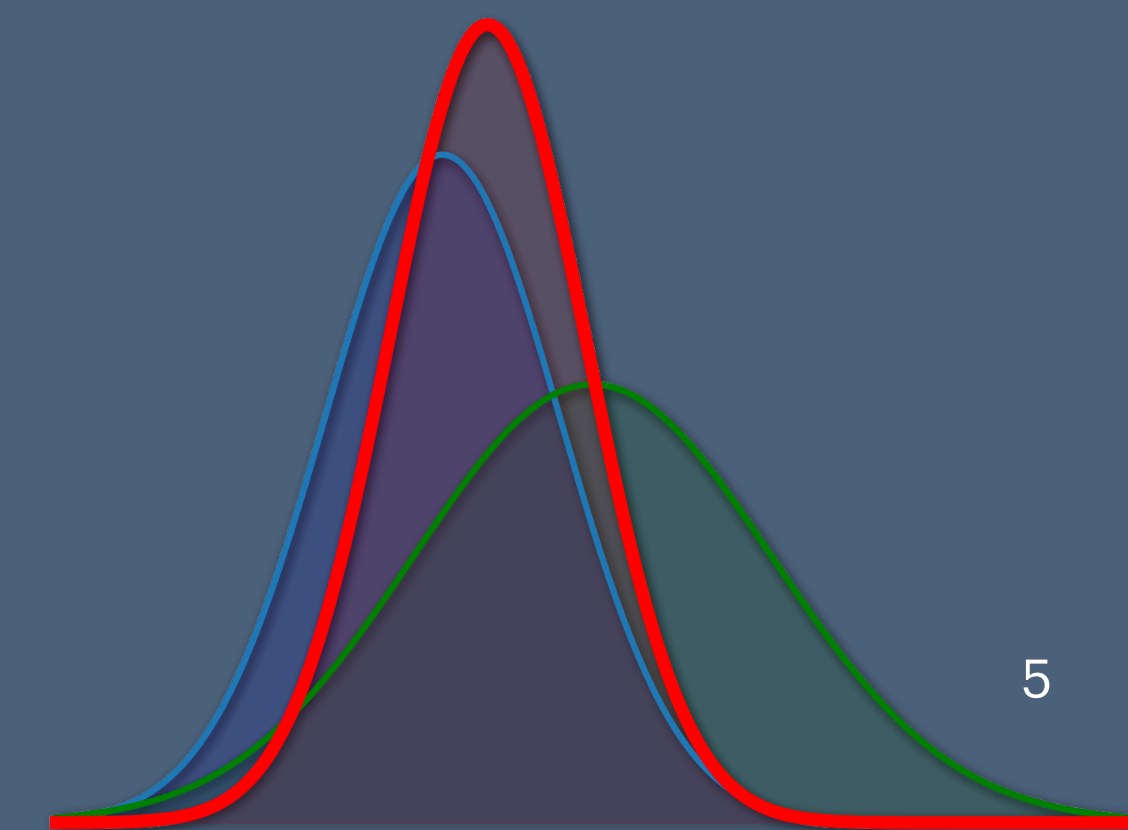
In the stochastic case, the solution of the inverse problem, “find u from y ,” is a **posterior probability density function** (ppdf).

Theorem (Bayes)

$$p(u|y) = \frac{p(y|u)p(u)}{p(y)},$$

or

$$p(\text{parameter}|\text{data}) \propto p(\text{data}|\text{parameter})p(\text{parameter}).$$

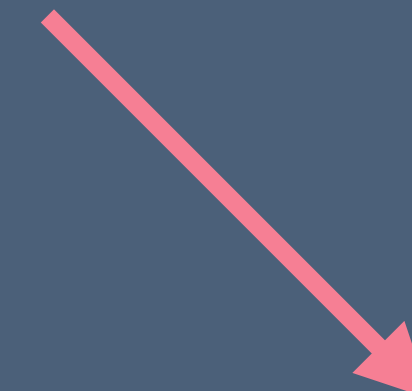


ML/AI in a Nutshell

Finding a pattern.

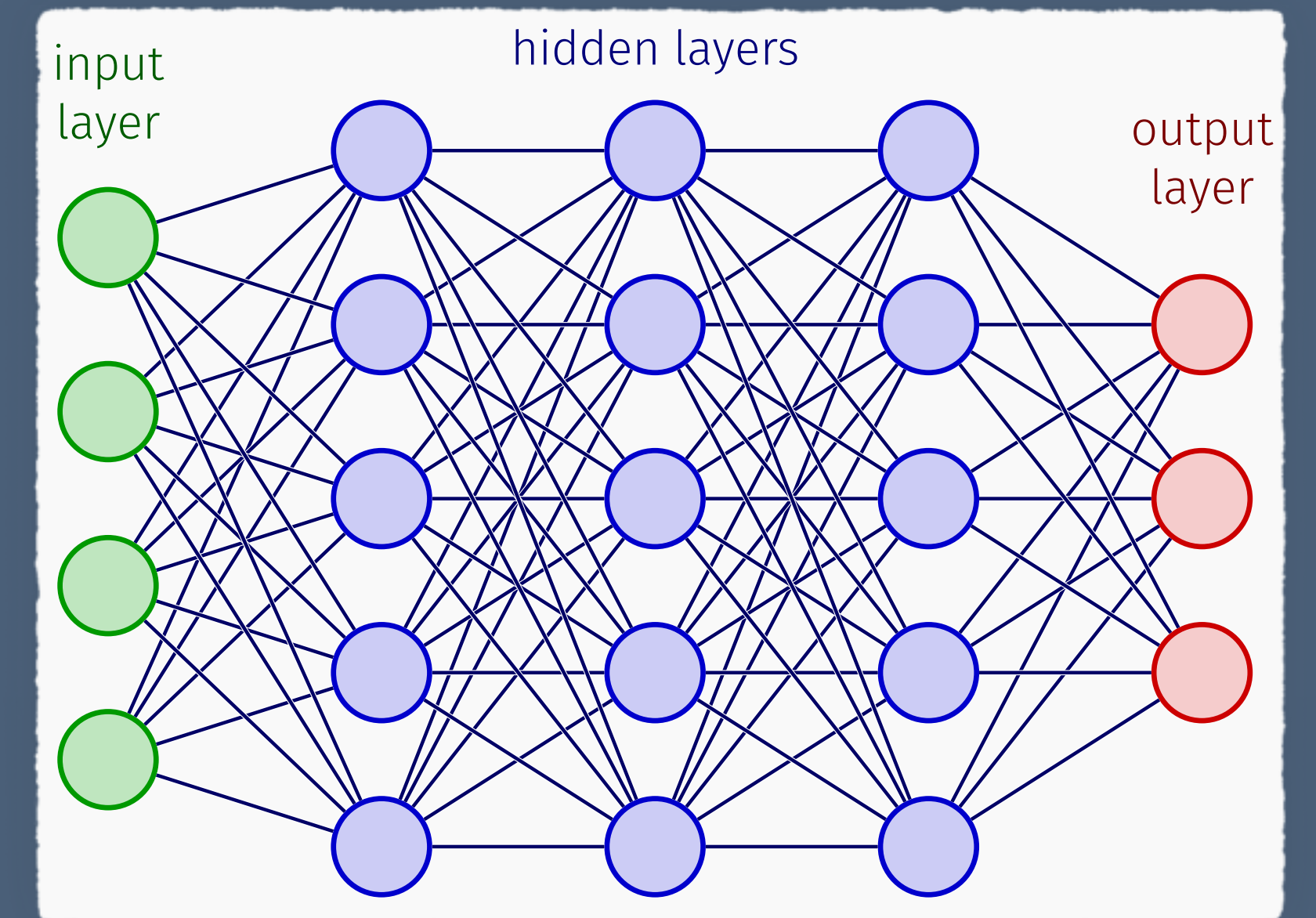
$$f : x \rightarrow y$$

- 
- Input
 - Features
 - Parameters
 - Causes

- 
- Output
 - Observations
 - Outcomes
 - Effects

ML/AI in a Nutshell

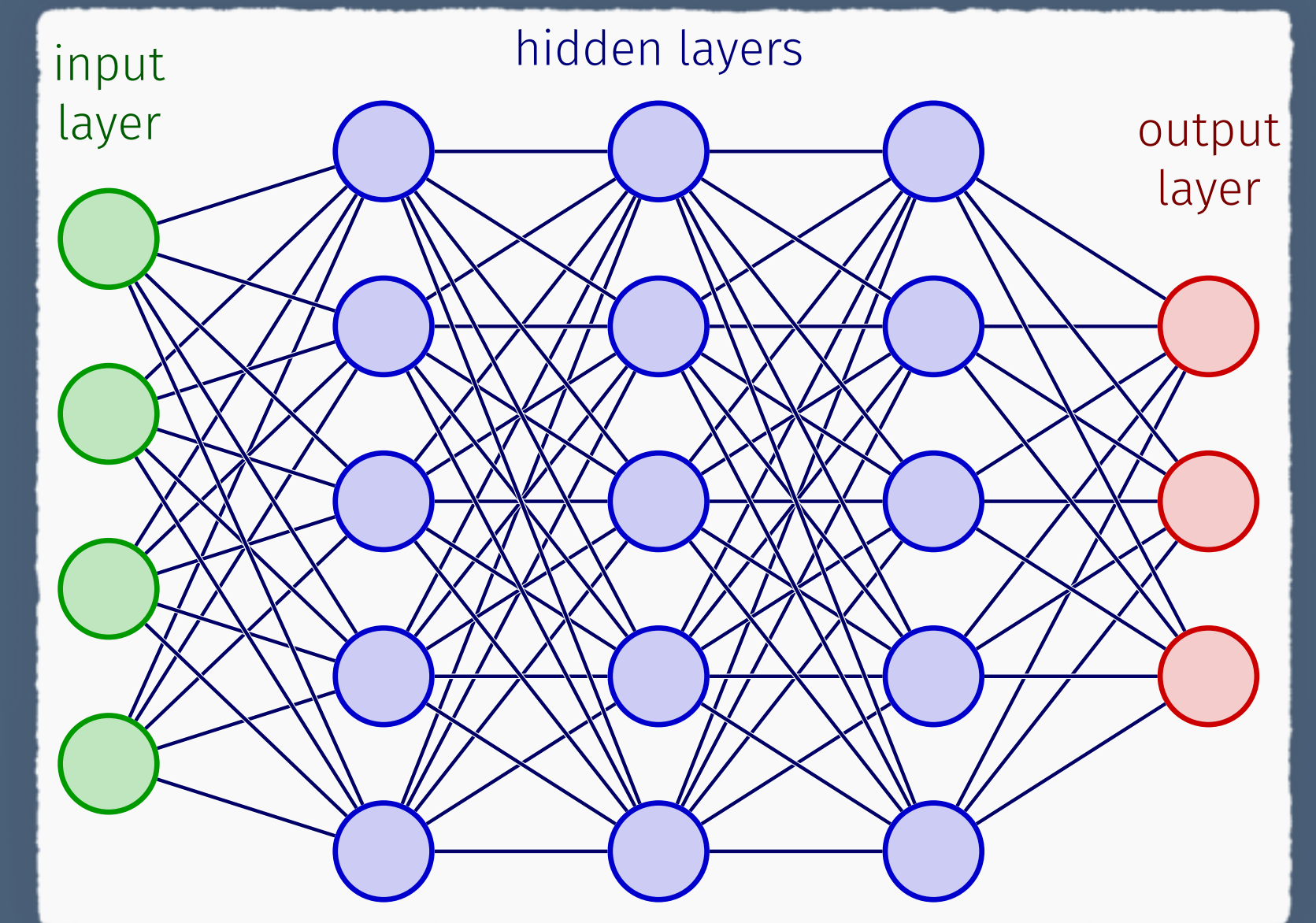
How is this done?



ML/AI in a Nutshell

How is this done?

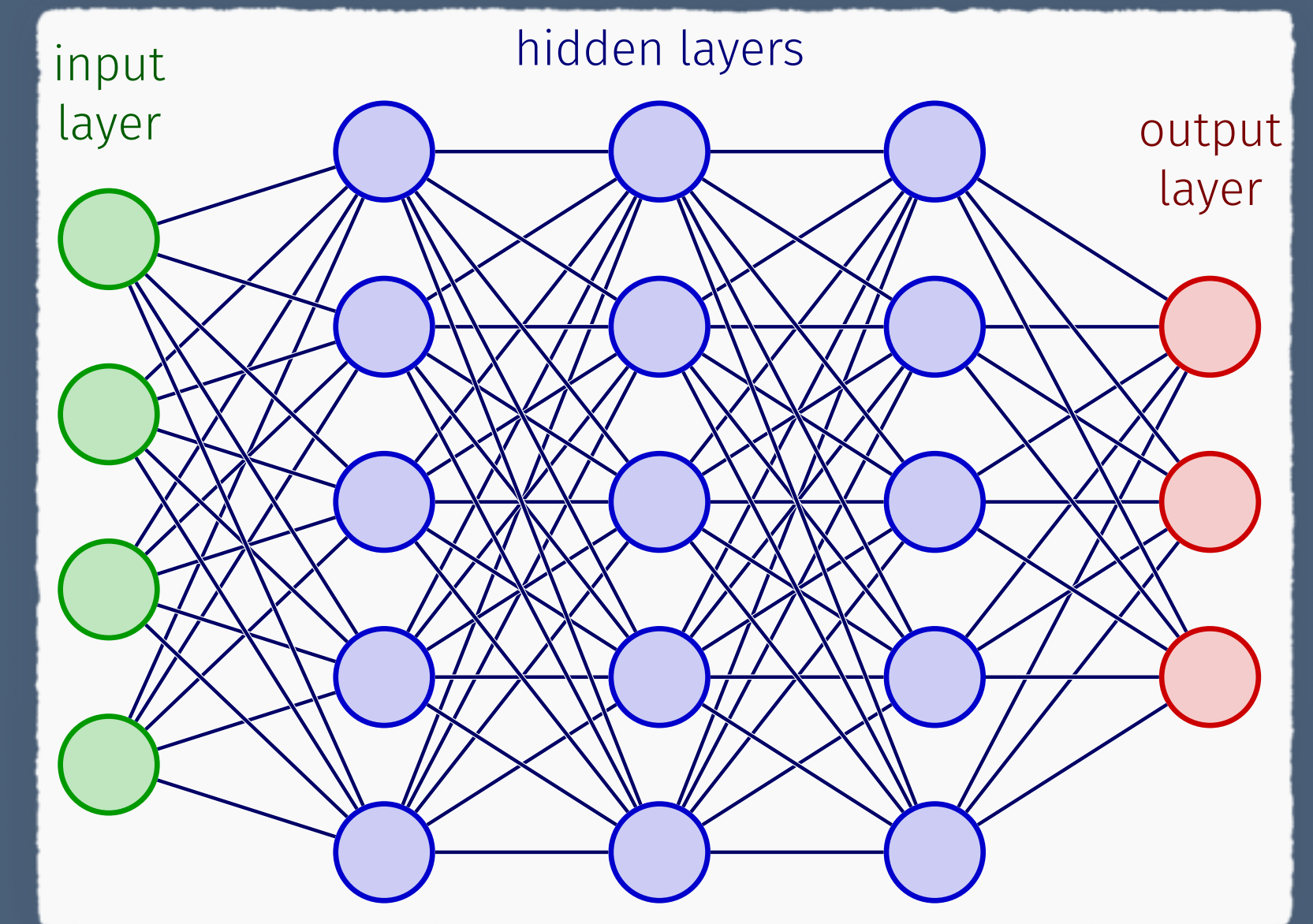
1. Suppose the ML/AI model depends on **parameters**, θ .



ML/AI in a Nutshell

How is this done?

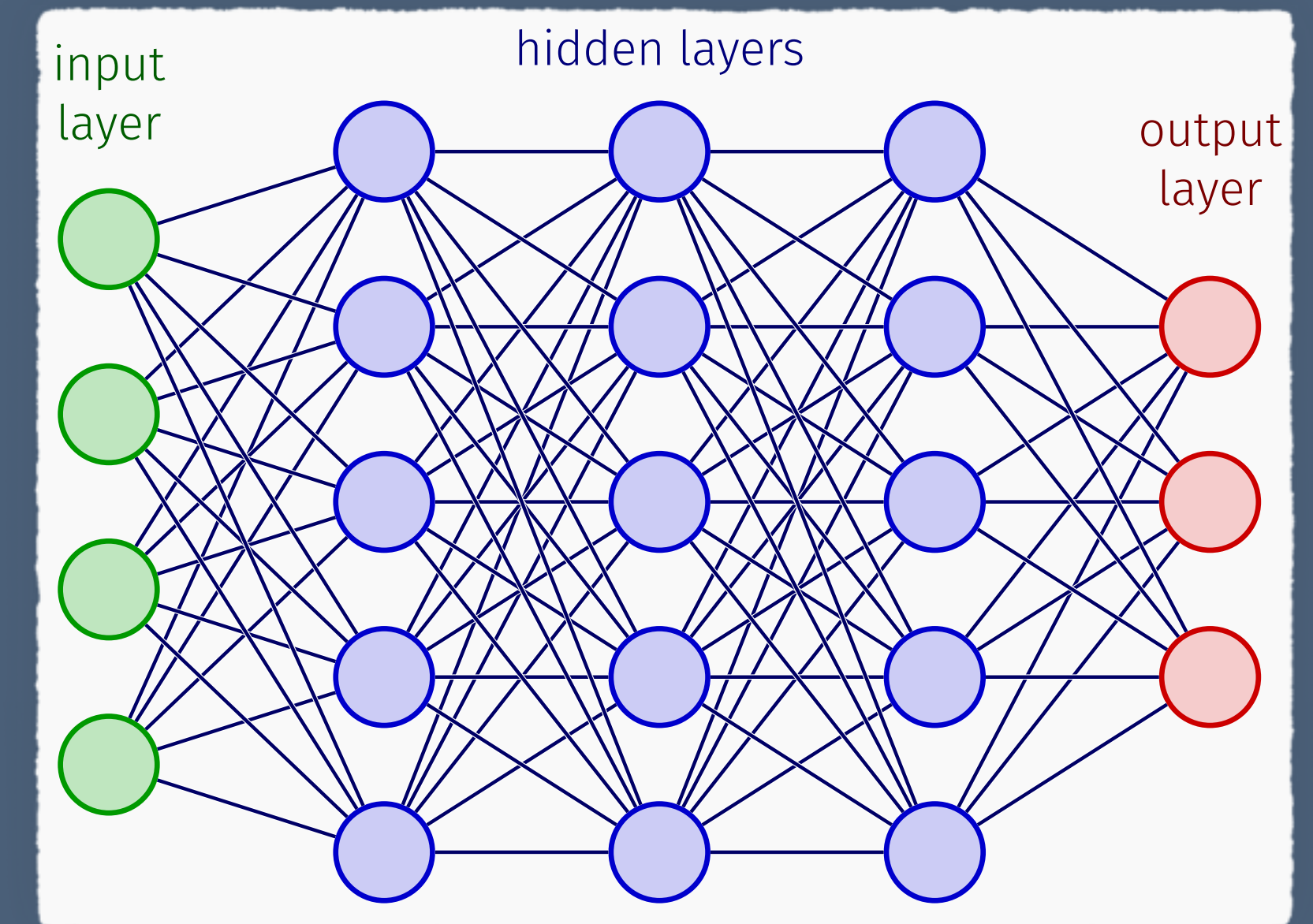
1. Suppose the ML/AI model depends on **parameters**, θ .
2. Define a **loss function** $\mathcal{L}$ that measures the difference between model predictions and measured observations (eg. MSE, cross-entropy).



ML/AI in a Nutshell

How is this done?

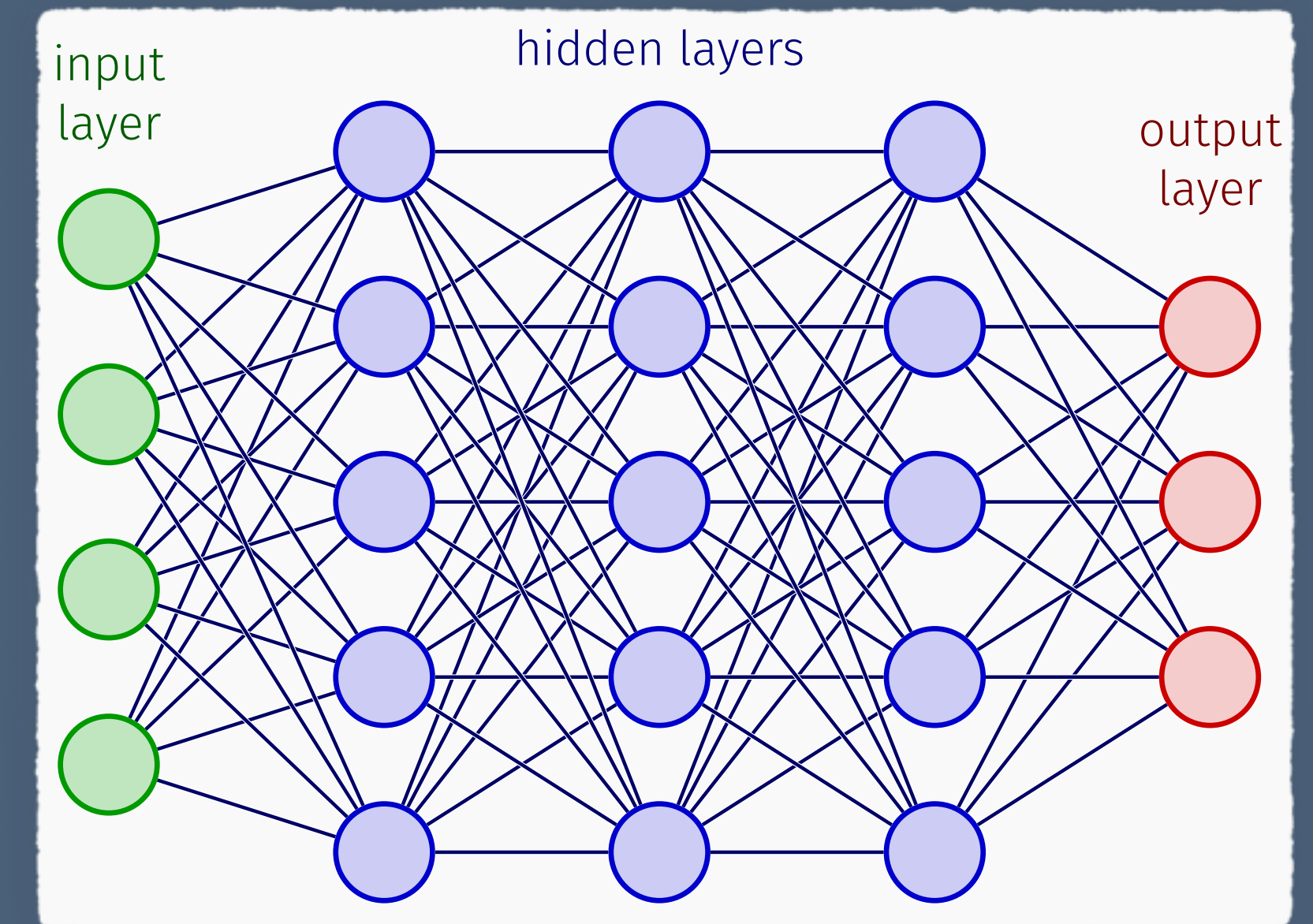
1. Suppose the ML/AI model depends on **parameters**, θ .
2. Define a **loss function** $\mathcal{L}$ that measures the difference between model predictions and measured observations (eg. MSE, cross-entropy).
3. Find the optimal parameters θ^* of the ML model that **minimize** the loss function.



ML/AI in a Nutshell

How is this done?

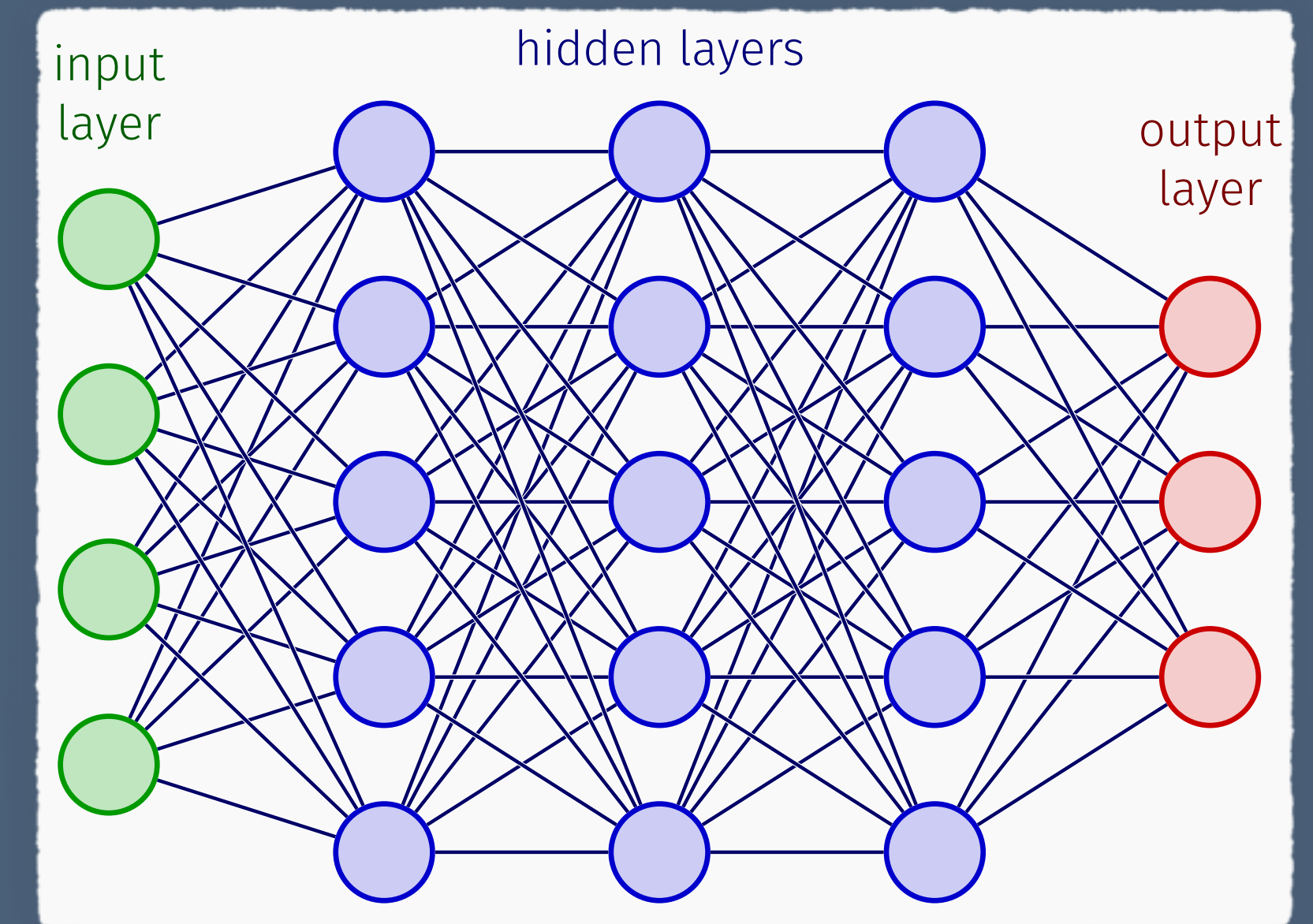
1. Suppose the ML/AI model depends on **parameters**, θ .
2. Define a **loss function** $\mathcal{L}$ that measures the difference between model predictions and measured observations (eg. MSE, cross-entropy).
3. Find the optimal parameters θ^* of the ML model that **minimize** the loss function.
 - *This is nothing more than an **inverse problem**.*



ML/AI in a Nutshell

How is this done?

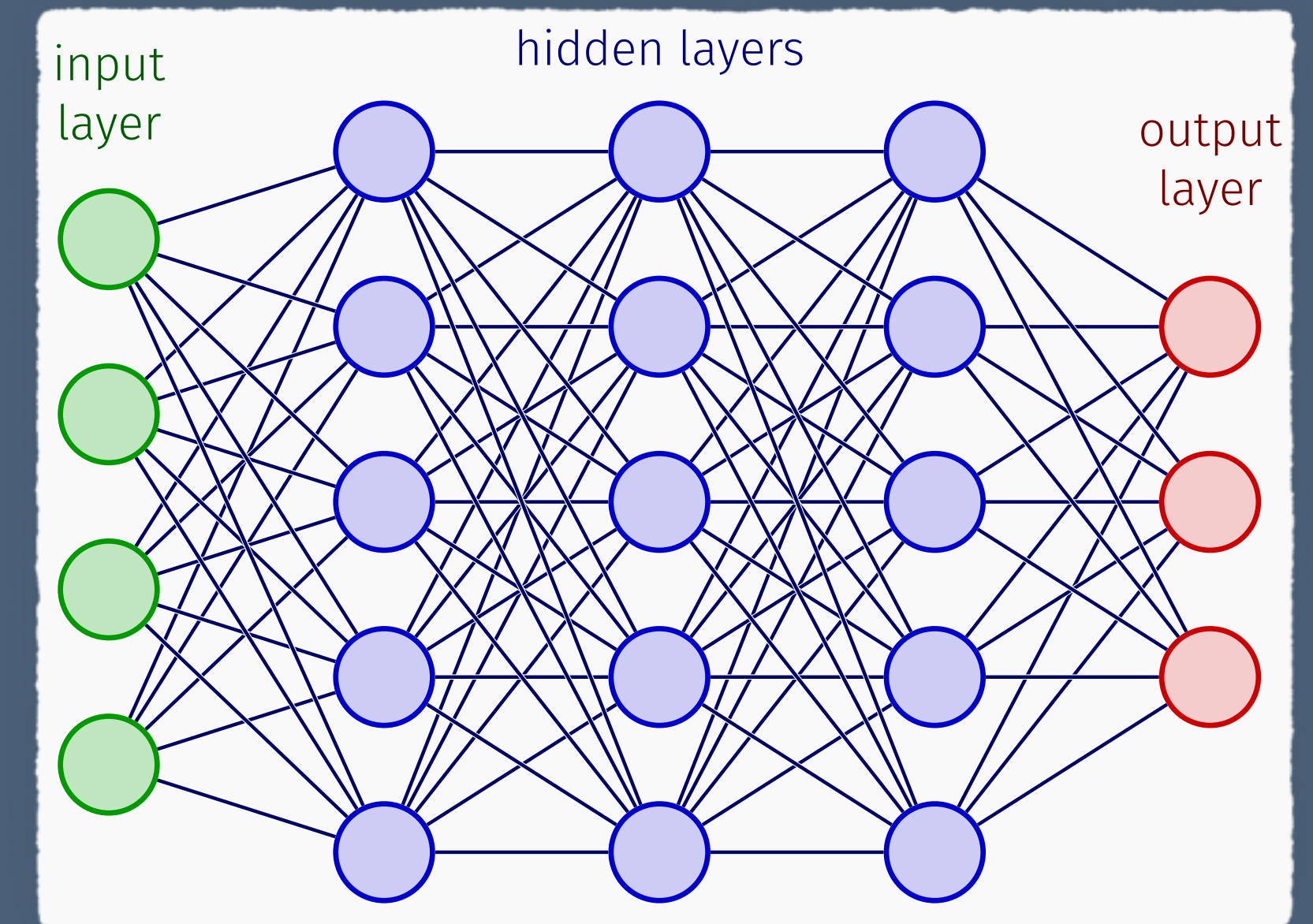
1. Suppose the ML/AI model depends on **parameters**, θ .
2. Define a **loss function** $\mathcal{L}$ that measures the difference between model predictions and measured observations (eg. MSE, cross-entropy).
3. Find the optimal parameters θ^* of the ML model that **minimize** the loss function.
 - *This is nothing more than an **inverse problem**.*
 - *Inverse problems are at the core of **Digital Twins**.*



ML/AI in a Nutshell

How is this done?

1. Suppose the ML/AI model depends on **parameters**, θ .
2. Define a **loss function** $\mathcal{L}$ that measures the difference between model predictions and measured observations (eg. MSE, cross-entropy).
3. Find the optimal parameters θ^* of the ML model that **minimize** the loss function.
 - *This is nothing more than an **inverse problem**.*
 - *Inverse problems are at the core of **Digital Twins**.*
 - ***Scientific ML** will improve certifiability—see below.*

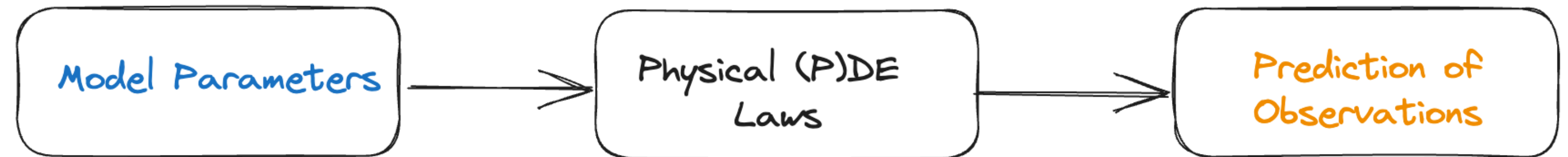


Inverse Problems

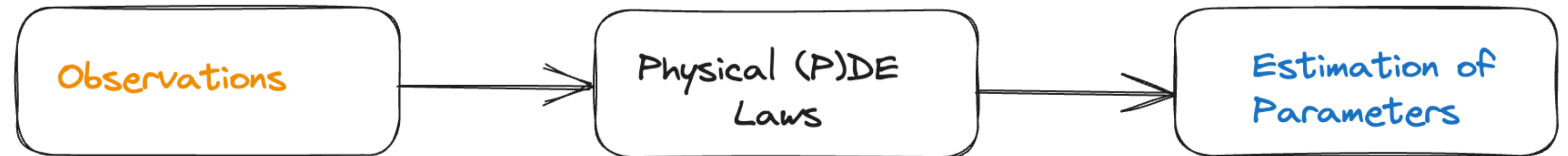
Classes

- Linear - Nonlinear
- Deterministic - Statistical
- Static - Dynamic
- Well-posed - Ill-posed

Direct Problem:



Inverse Problem:



Inverse Problems

Deterministic

- Given a physical relation

$$F(u; \boldsymbol{\theta}) = 0 \quad (2)$$

represented by an IBVP, or other functional relationship, with

⇒ u the physical quantity

⇒ $\boldsymbol{\theta}$ the (material/medium) properties/parameters

- **Inverse Problem** is defined as:

⇒ **Given** **observations**/measurements of u at the locations $\mathbf{x} = \{x_i\}$

$$\mathbf{u}^{\text{obs}} = \{u(x_i)\}_{i \in \mathcal{I}}$$

⇒ **Estimate** the **parameters** $\boldsymbol{\theta}$ by minimizing a loss/objective/cost function

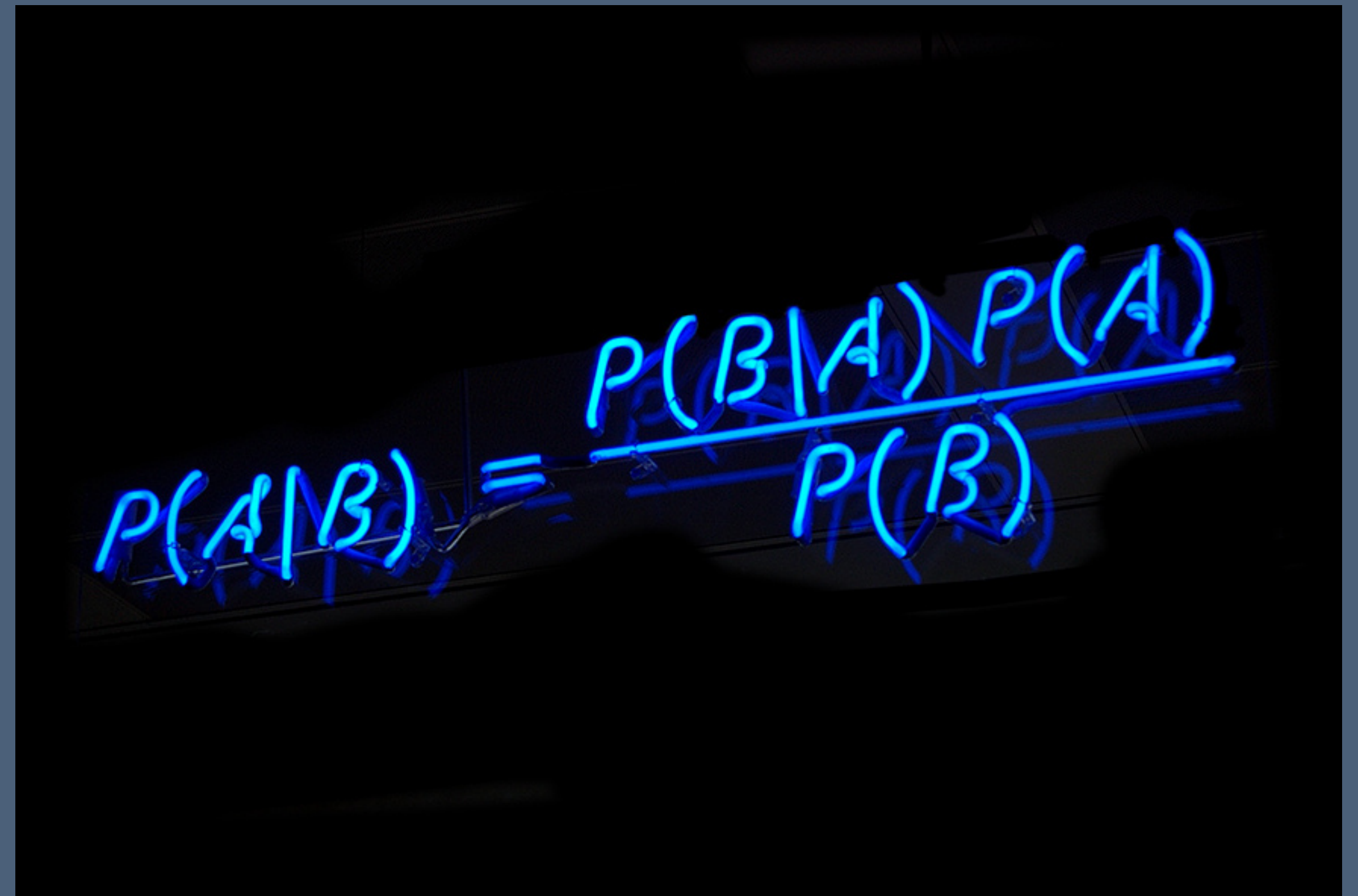
$$L(\boldsymbol{\theta}) = \|u(\mathbf{x}) - \mathbf{u}^{\text{obs}}\|_2^2$$

subject to (2).

Inverse Problems

Statistical (Stochastic)

- Idea: reformulate inverse problems as problems of **statistical inference** by means of **Bayesian** statistics.
 - All quantities and parameters are modeled as **random variables**.
 - Ill-posed IP becomes **well-posed** in this setting!
- From the perspective of **statistical inversion theory**, the solution to an inverse problem is the probability distribution of the quantity of interest when all information available has been incorporated in the model.
- This distribution, the **posterior distribution**, describes the degree of confidence about the quantity of interest, after the measurement has been performed.



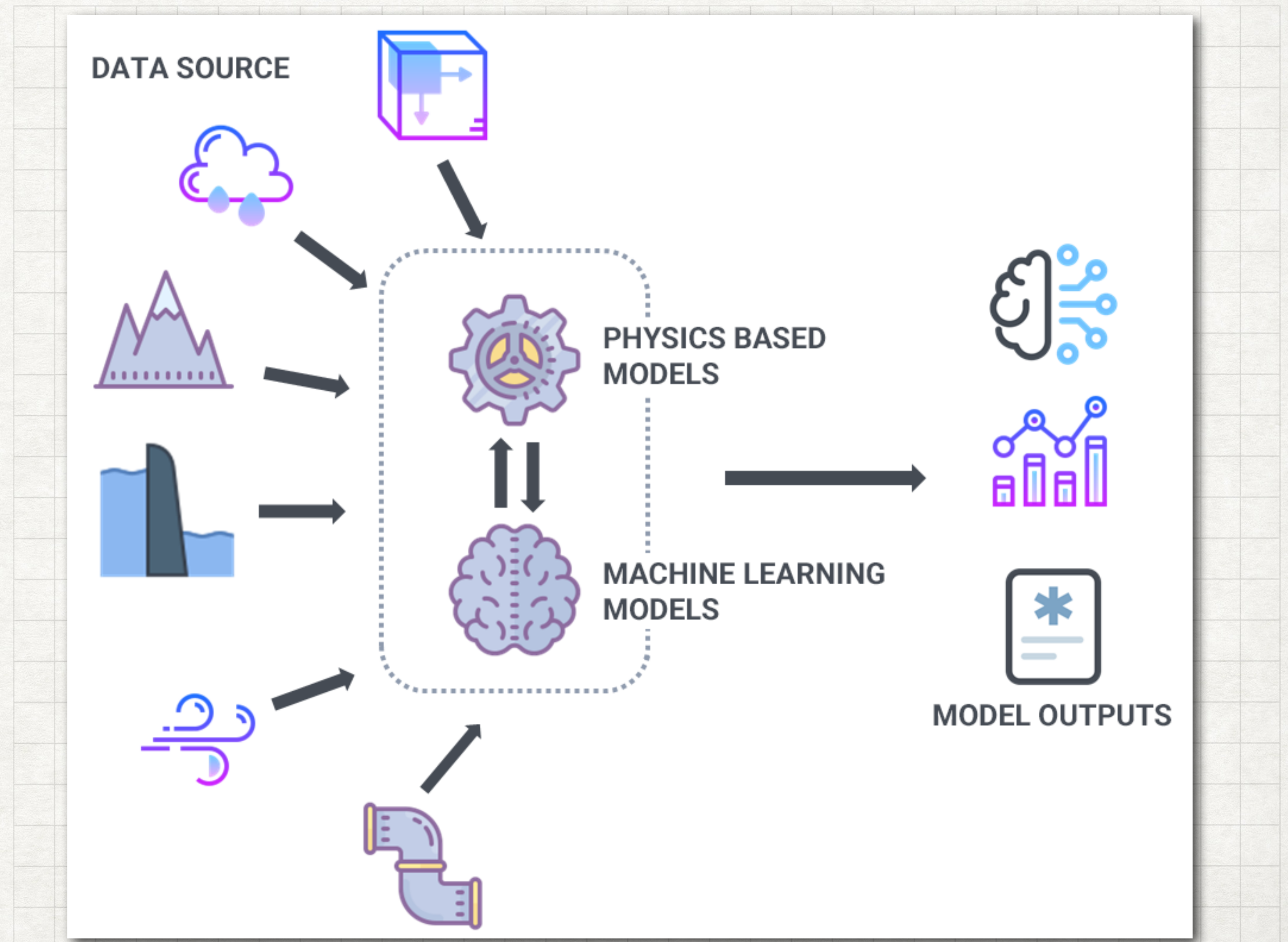
A photograph of a chalkboard with the Bayesian formula $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$ written in blue chalk. The formula is written in a slightly slanted, handwritten style. The background is dark, and the chalk is bright blue.

$$f(\text{parameter} \mid \text{data}) \propto f(\text{data} \mid \text{parameter})f(\text{parameter})$$

WHAT IS SCIENTIFIC ML?

TWO WORLDS UNITED

Scientific Machine Learning (SciML) is a field of research that combines traditional *scientific modeling* with *machine learning* techniques. It aims to develop new methods and tools for solving scientific problems that are more accurate, efficient, and generalizable than traditional methods.



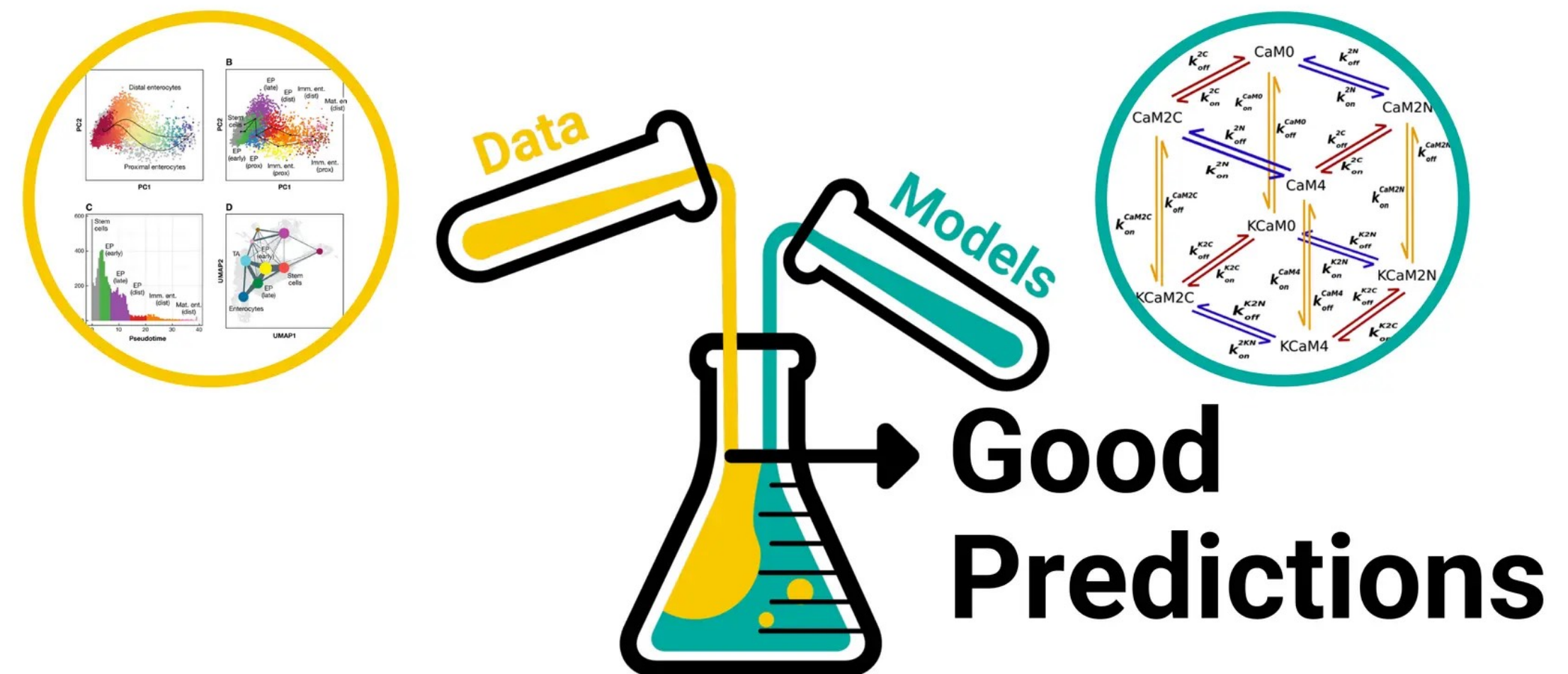
WHAT IS SCIENTIFIC ML?

BLENDING

- SciML should exploit any available, underlying **physical** knowledge/principles:
 - Laws, equations.
 - Conserved quantities.
 - Symmetries, etc.
- Improves **interpretability** and **certifiability**—no more black boxes!
- **Reduces** needs for vast amounts of training data.
- **Assists** in DL network design (theory-inspired).
- **Incorporates** ML models into physical plants/devices.

Scientific Machine Learning is model-based data-efficient machine learning

How do we simultaneously use both sources of knowledge?



[C. Rackaukas]

Scientific Machine Learning - relation with DTs

- ML and DTs are essentially **optimization** problems, where a loss/mismatch function is minimised.
 - This is just a norm of the difference between model output and measured data.
 - It generates very ill-posed problems (very high dimensional, noisy, etc.).
- But, ML does it extremely well (optimisation + regularisation) with amazing tools.
 - **Neural networks**, **Stochastic gradient** methods
 - **Back propagation** is equivalent to a classical adjoint method.
 - **Mini-batches** (don't try to minimise over all the variables at once), **dropout** (UQ).
- **Q: Does the ML solution respect the physics?**
- **A: Use SciML!**

SciML-Theory

- Recall: there are two possible **optimization strategies** for constraining the NN (ML) to respect the physics

1. Hard constraints
2. Soft constraints

- Suppose we have a (P)DE of the form

$$\mathcal{F}(u(\mathbf{x}, t)) = 0, \quad \mathbf{x} \in \Omega \subset \mathbb{R}^d, \quad t \in [0, T],$$

where

- $\Rightarrow \mathcal{F}$ is a differential operator representing the (P)DE
- $\Rightarrow u(\mathbf{x}, t)$ is the state variable (i.e., quantity of interest), with $\mathbf{x}, t$ the space-time variables
- $\Rightarrow T$ is the time horizon and Ω is the spatial domain (empty for ODEs)
- $\Rightarrow$ initial and boundary conditions must be added for the problem to be well-posed

- **Hard constraint:** solve the constrained optimization problem

$$\min_{\theta} \mathcal{L}(u) \quad \text{s.t.} \quad \mathcal{F}(u) = 0,$$

where

- $\Rightarrow \mathcal{L}(u)$ is the data (mismatch) loss term
- $\Rightarrow \mathcal{F}$ is the constraint on the residual of the (P)DE under consideration
- $\Rightarrow$ as was amply discussed in the DA/inverse problem context, this type of (P)DE-constrained optimization is usually quite difficult to code and to solve

- **Soft constraint:** solve the regularized/penalized unconstrained optimization problem

$$\min_{\theta} \mathcal{L}(u) + \alpha_{\mathcal{F}} \mathcal{F}(u), \quad (3)$$

$$\mathcal{L}(u) = \mathcal{L}_{u_0} + \mathcal{L}_{u_b},$$

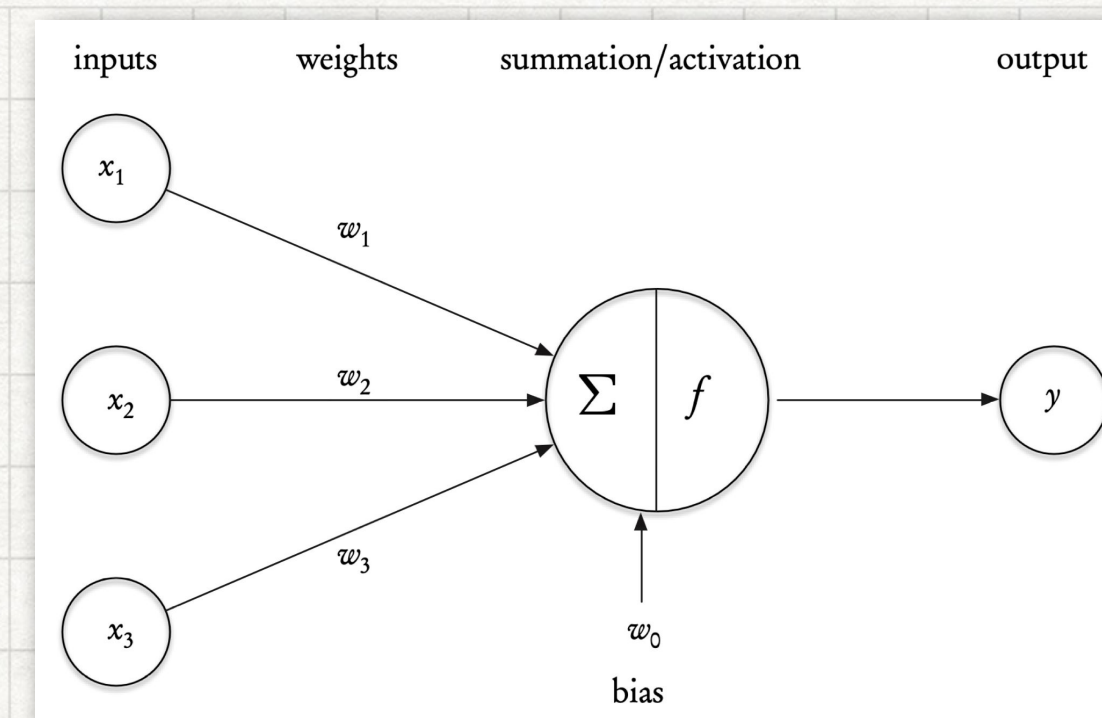
where

- $\Rightarrow \mathcal{L}_{u_0}$ represents the misfit of the NN predictions
- $\Rightarrow \mathcal{L}_{u_b}$ represents the misfit of the initial/boundary conditions
- $\Rightarrow \theta$ represents the NN parameters
- $\Rightarrow \alpha_{\mathcal{F}}$ is a regularization parameter that controls the emphasis on the PDE-based residual (which we ideally want to be zero)

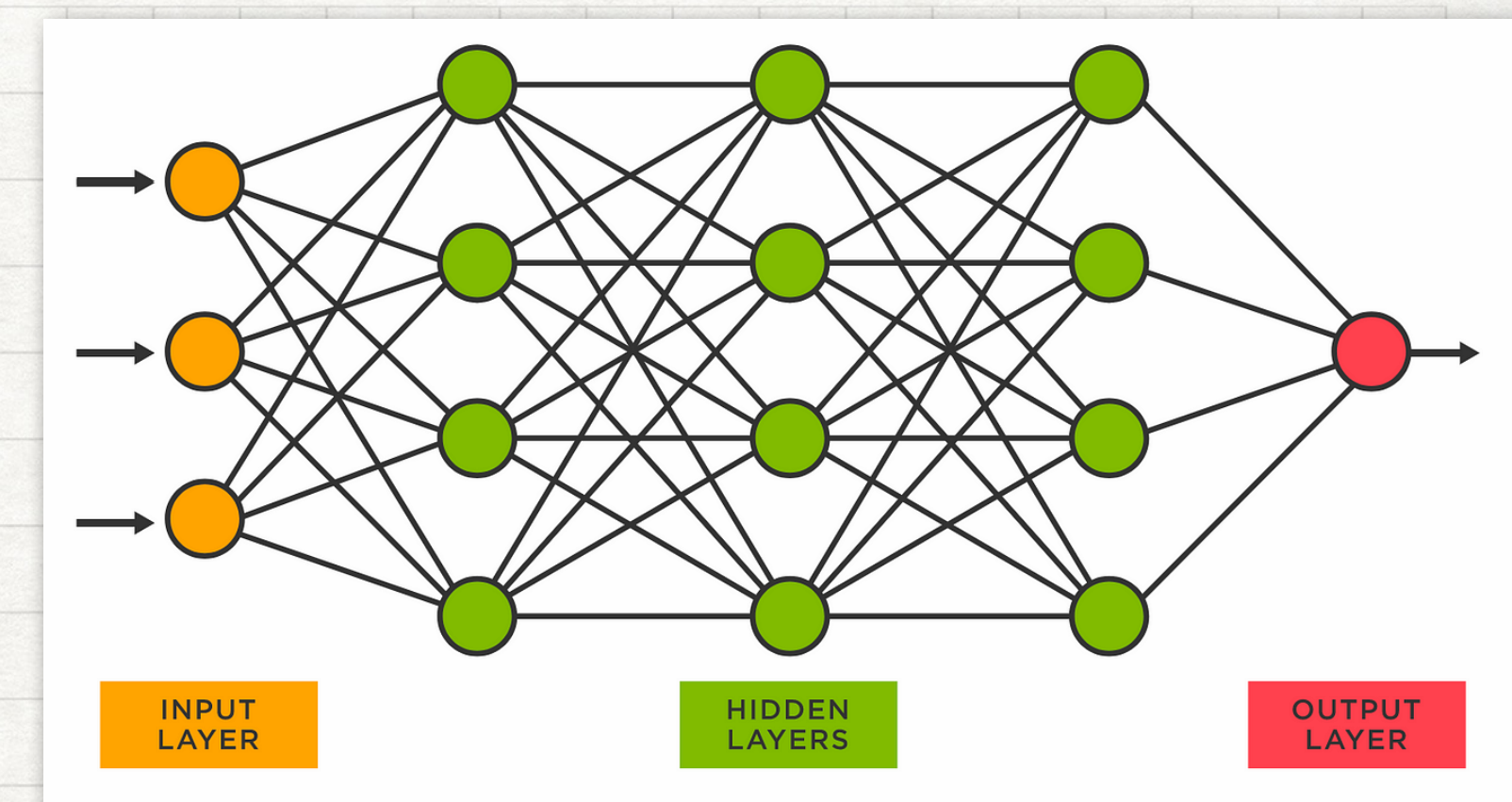
THE MATH BEHIND SCI-ML

APPROXIMATION THEORY

- Multi-layer perceptrons - 1950's - the basis.
- Universal Approximation Property - 1990's - the theory.



$$y = f\left(w_0 + \sum_{i=1}^3 w_i x_i\right)$$



Theorem 1 (Cybenko 1989). *If σ is any continuous sigmoidal function, then finite sums*

$$G(x) = \sum_{j=1}^N \alpha_j \sigma(w_j \cdot x + b_j)$$

*are **dense** in $C(I_d)$.*

Theorem 2 (Pinkus 1999). *Let $\mathbf{m}_i \in \mathbb{Z}^d$, $i = 1, \dots, s$, and set $m = \max_i |\mathbf{m}^i|$. Suppose that $\sigma \in C^m(\mathbb{R})$, not polynomial. Then the space of **single hidden layer** neural nets,*

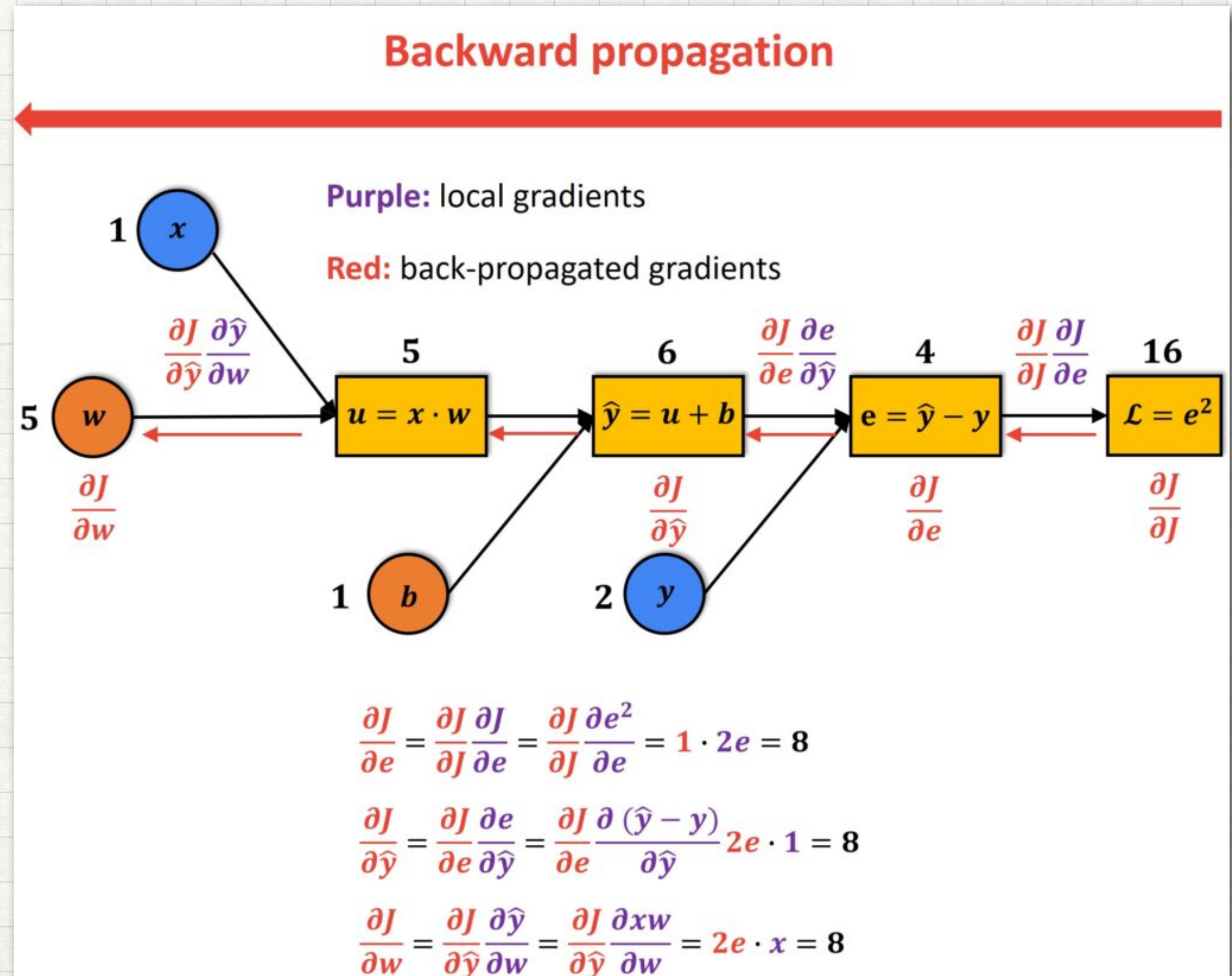
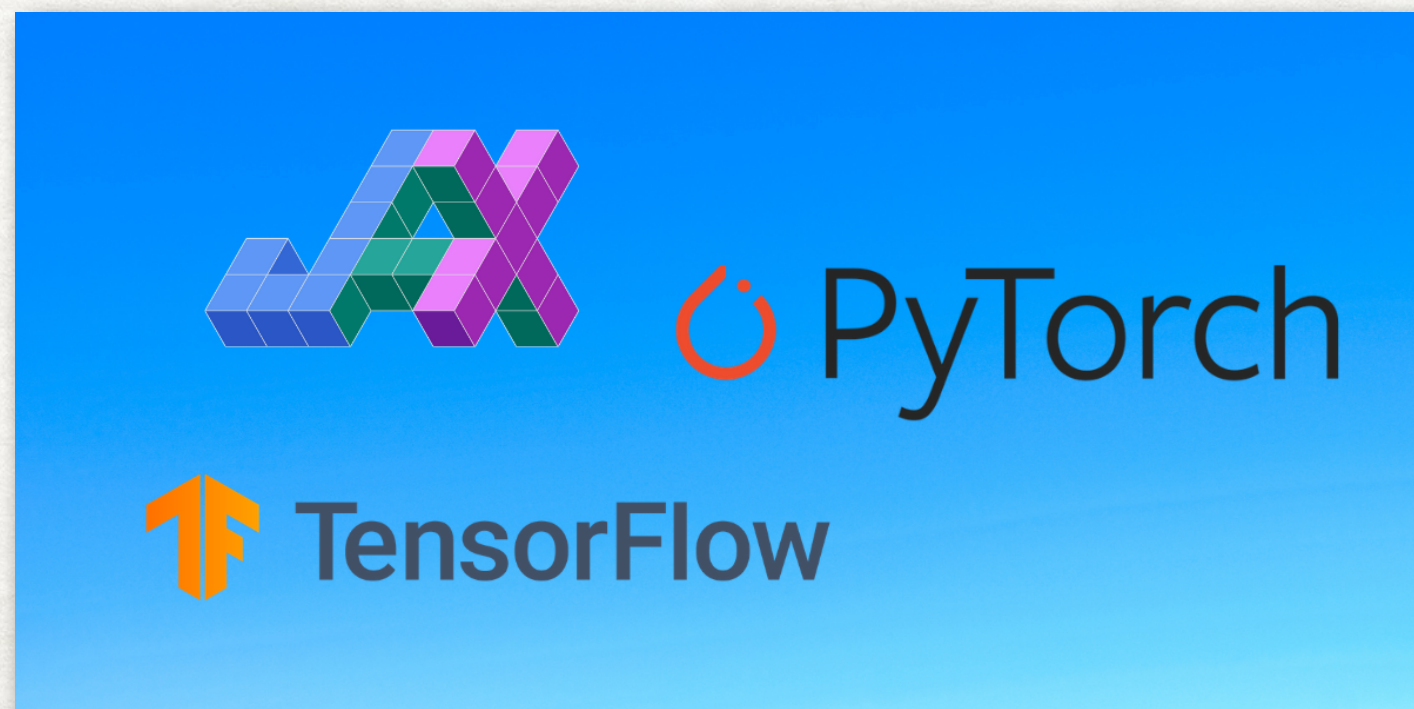
$$\mathcal{M}(\sigma) = \text{span} \{ \sigma(\mathbf{w} \cdot \mathbf{x} + b) : \mathbf{w} \in \mathbb{R}^d, b \in \mathbb{R} \},$$

is dense in $C^{\mathbf{m}^1, \dots, \mathbf{m}^s}(\mathbb{R}^d) \doteq \cap_{i=1}^s C^{\mathbf{m}^i}(\mathbb{R}^d)$.

THE CODE BEHIND SCI-ML

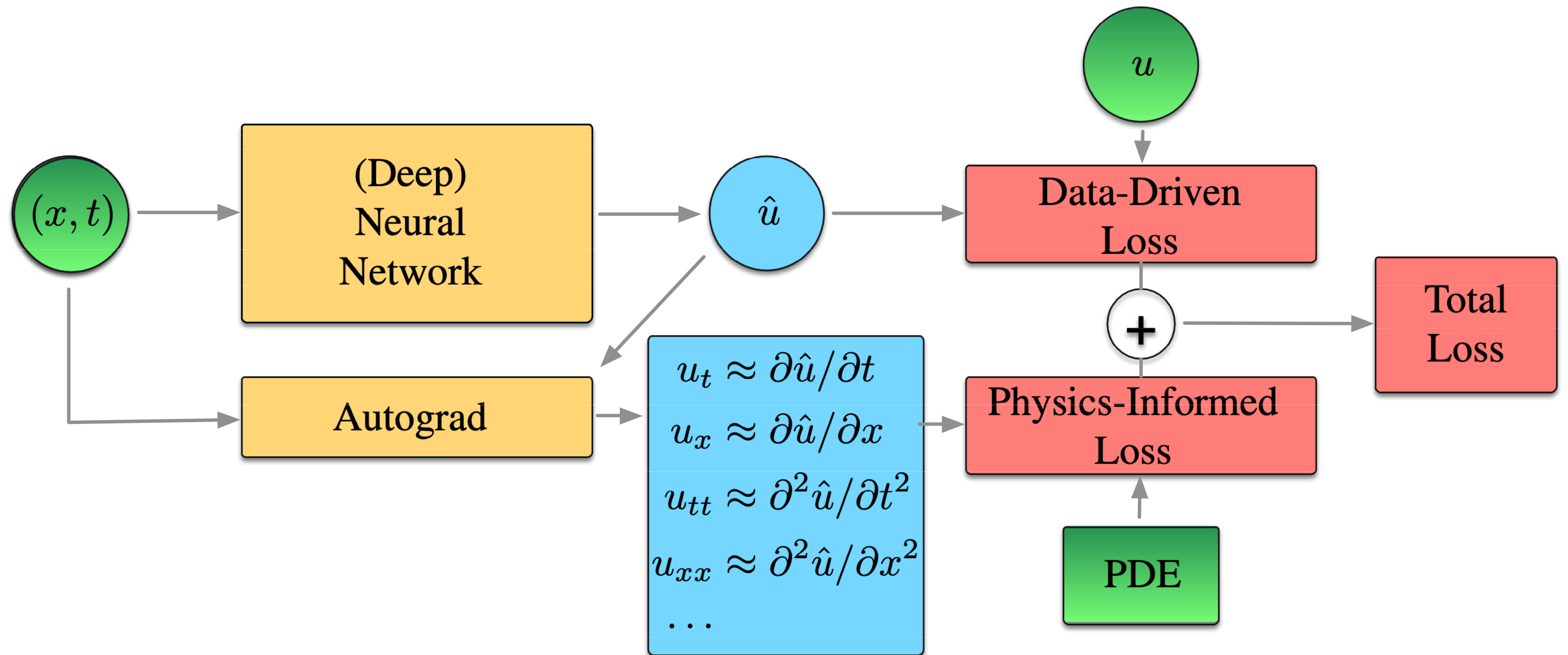
"AUTOGRAD"

- Differentiable programming - 2020's
 - makes it all possible!
- Based on:
 - Computational graphs
 - Chain-rule for differentiation



PINN

For any ODE/PDE



PINN Example

Example: IBVP for Diffusion Equation

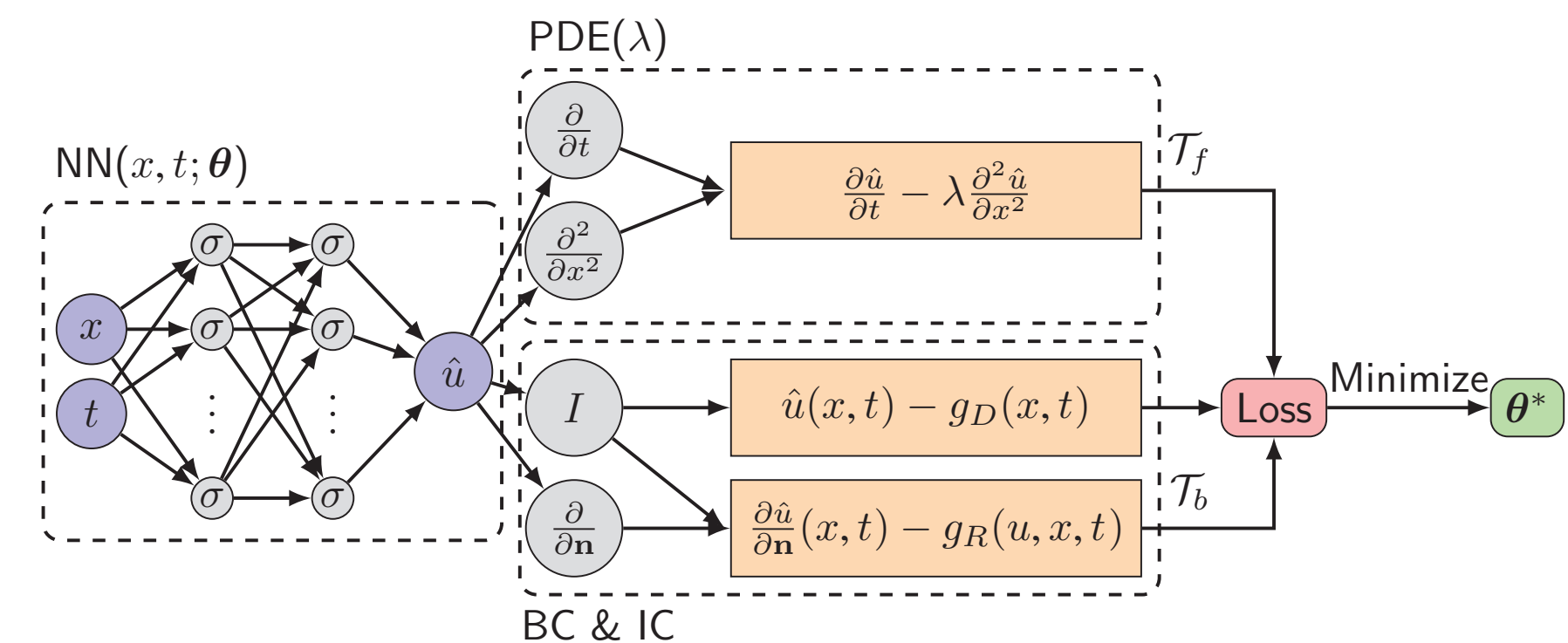
Compute $u(\mathbf{x}, t): \Omega \times [0, T] \rightarrow \mathbb{R}$ such that

$$\begin{aligned} \frac{\partial u(\mathbf{x}, t)}{\partial t} - \nabla \cdot (\lambda(x) \nabla u(\mathbf{x}, t)) &= f(\mathbf{x}, t) \quad \text{in } \Omega \times (0, T), \\ u(\mathbf{x}, t) &= g_D(\mathbf{x}, t) \quad \text{on } \partial\Omega_D \times (0, T), \\ -\lambda(x) \nabla u(\mathbf{x}, t) \cdot \mathbf{n} &= g_R(\mathbf{x}, t) \quad \text{on } \partial\Omega_R \times (0, T), \\ u(\mathbf{x}, 0) &= u_0(\mathbf{x}) \quad \text{for } \mathbf{x} \in \Omega. \end{aligned} \quad (6)$$

Note that $\lambda(x)$ is, in general, a tensor (matrix) with elements λ_{ij} .

- **Direct problem:** given λ , compute u .
- **Inverse problem:** given u , compute λ .

PINN for the Diffusion Equation



[Credit: Lu, Karniadakis, SIAM Review, 2021]

- Use **FCNN** to approximate u at the selected points x , with training data at residual points $\mathcal{T}_f$ and $\mathcal{T}_b$
- Use **AD** to compute derivatives for the PDE and the boundary/initial conditions
- **Minimize** the augmented, weighted loss function

HOW IS SCIENTIFIC ML DONE?

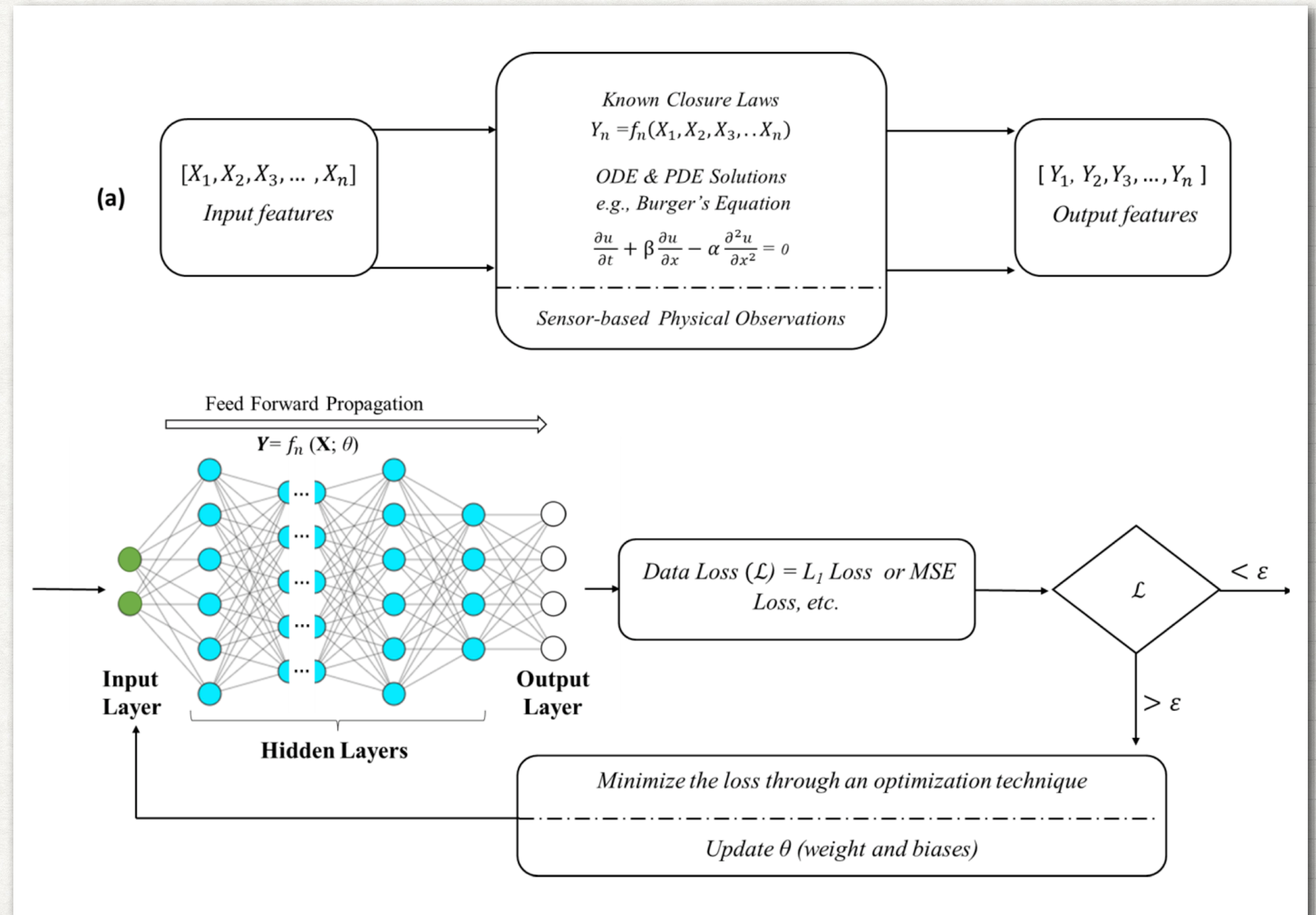
BLENDING

3 possible paths:

1. Physics-guided NNs

2. Physics-informed NNs

3. Physics-encoded NNs

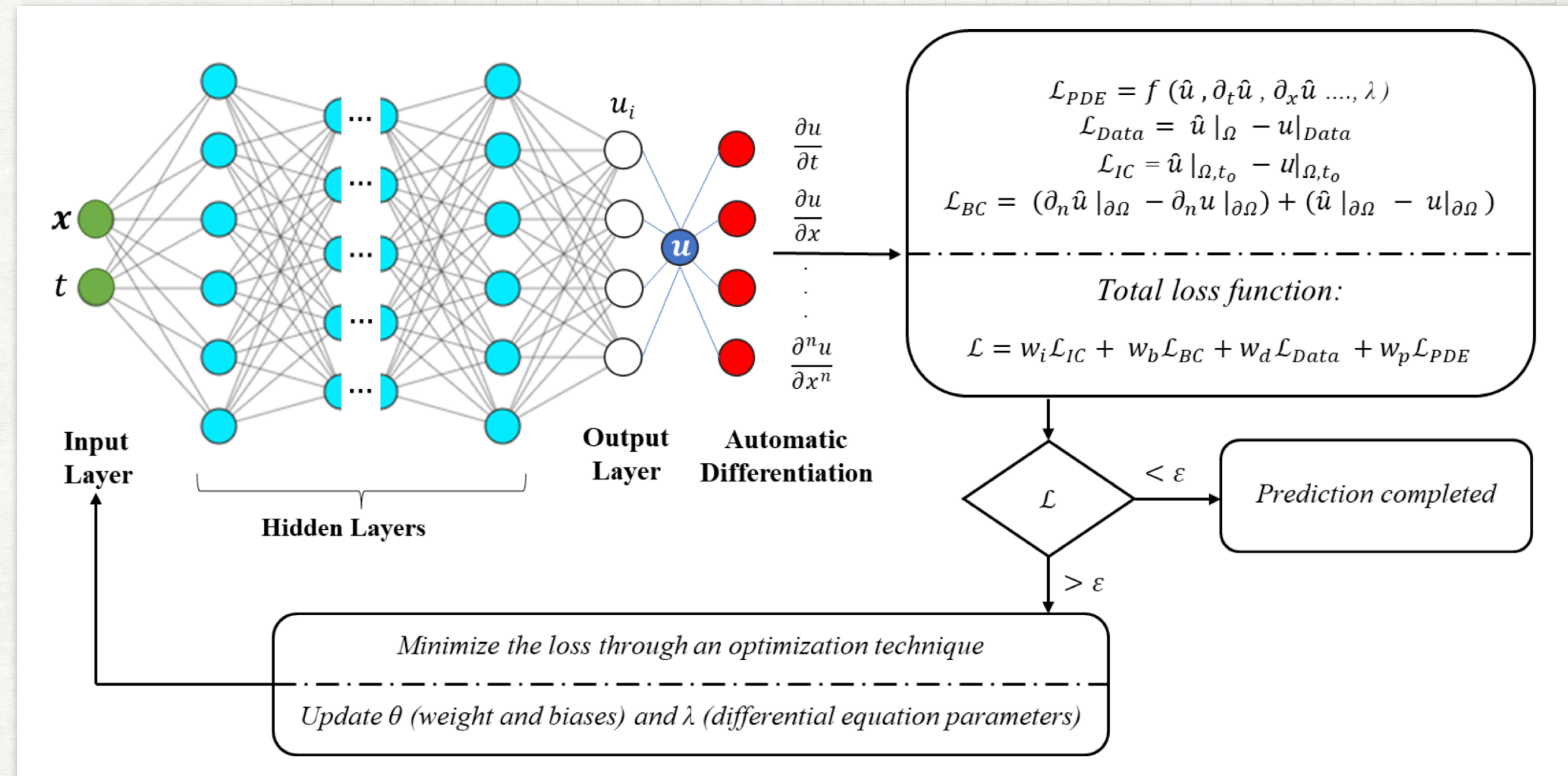


HOW IS SCIENTIFIC ML DONE?

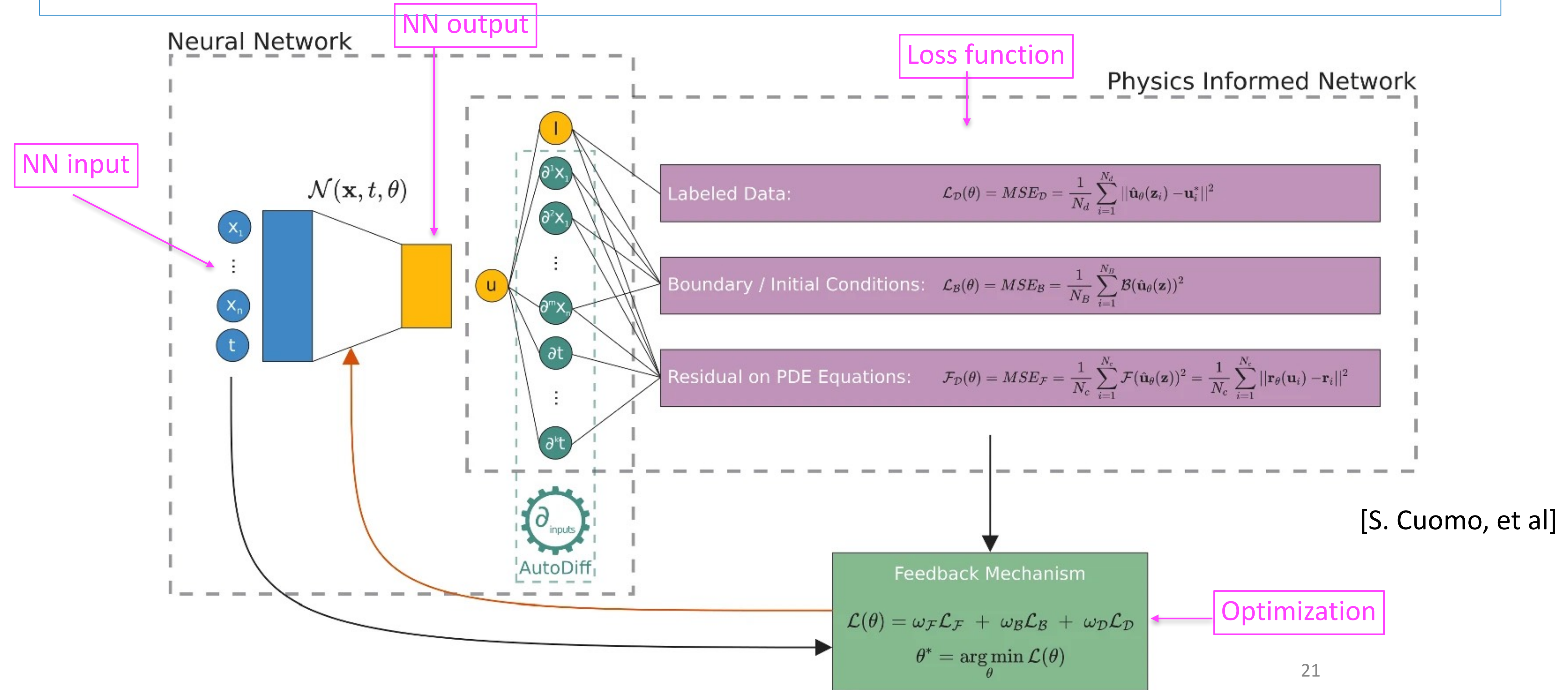
BLENDING

3 possible paths:

1. Physics-guided NNs
2. Physics-informed NNs
3. Physics-encoded NNs



SciML - PINN - Blending

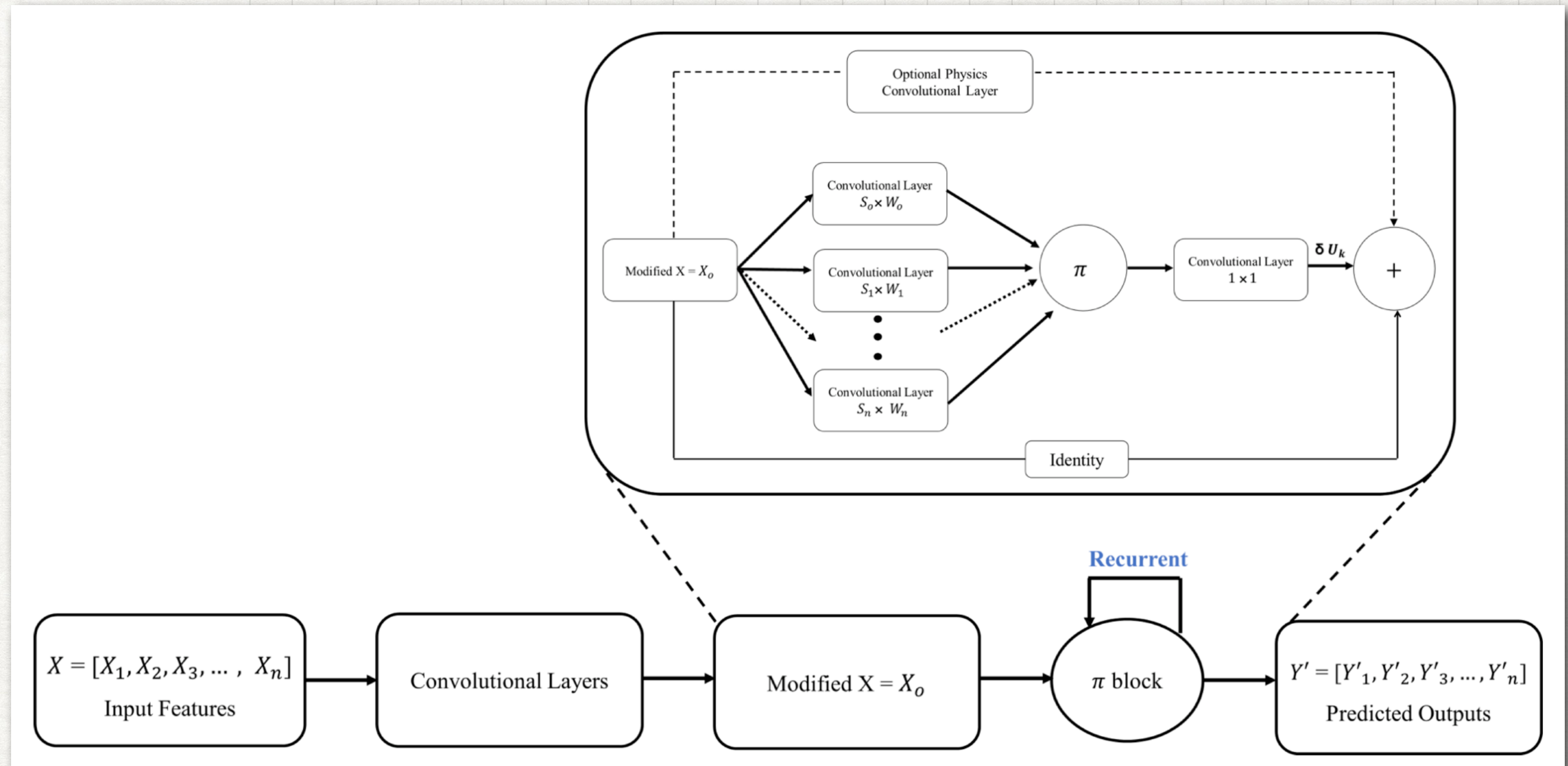


HOW IS SCIENTIFIC ML DONE?

BLENDING

3 possible paths:

1. Physics-guided NNs
2. Physics-informed NNs
3. Physics-encoded NNs



Examples & Applications

Domains in biomedical sciences

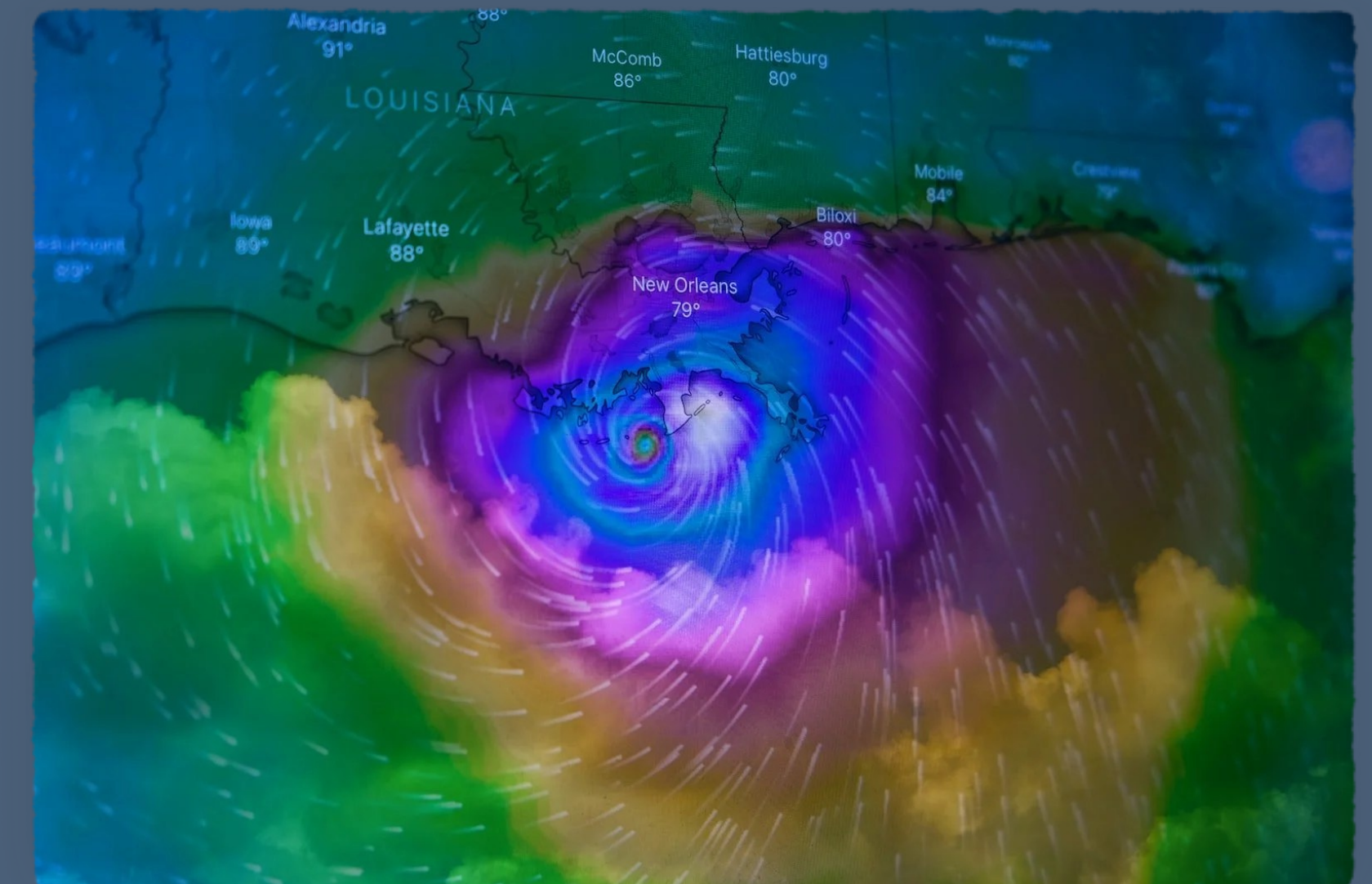
- Epidemiology
 - Omics
 - Medicine
 - Drug design
- COVID,
 - Anthrax,
 - HIV,
 - Zika,
 - Smallpox,
 - Tuberculosis,
 - Pneumonia,
 - Ebola,
 - Dengue,
 - Polio,
 - Measles.

Epidemiology with Scientific machine learning

- ML can be employed to study **complex interactions** between different biological systems, such as signaling pathways and metabolic networks, to advance our understanding of various biological phenomena and improve the diagnosis and treatment of diseases. These technologies have the potential to significantly impact biological research in a variety of areas, including infectious diseases and epidemiology.
- ML can be used to **analyze large datasets**, such as genomic data, to identify patterns and trends relevant to the understanding and treatment of infectious diseases
- ML can be employed to predict the **likelihood of certain outcomes**, such as the spread of a disease, based on historical data and by analyzing datasets generated by epidemiological studies. This can aid epidemiologists in preventing or mitigating outbreaks of infectious diseases, such as influenza and HIV.
- AI-based DTs can also be utilized to build **predictive models** that help researchers understand the relationships between different variables, such as gene expression and disease risk, interactions between pathogens and host organisms at the molecular level, and complex molecular interactions within biomolecules.

Other Scientific Domains

- Fluid dynamics, flow problems
 - Oceanography
 - Meteorology
 - Geophysical flows
- Aerodynamics
 - Solid mechanics
 - Material science

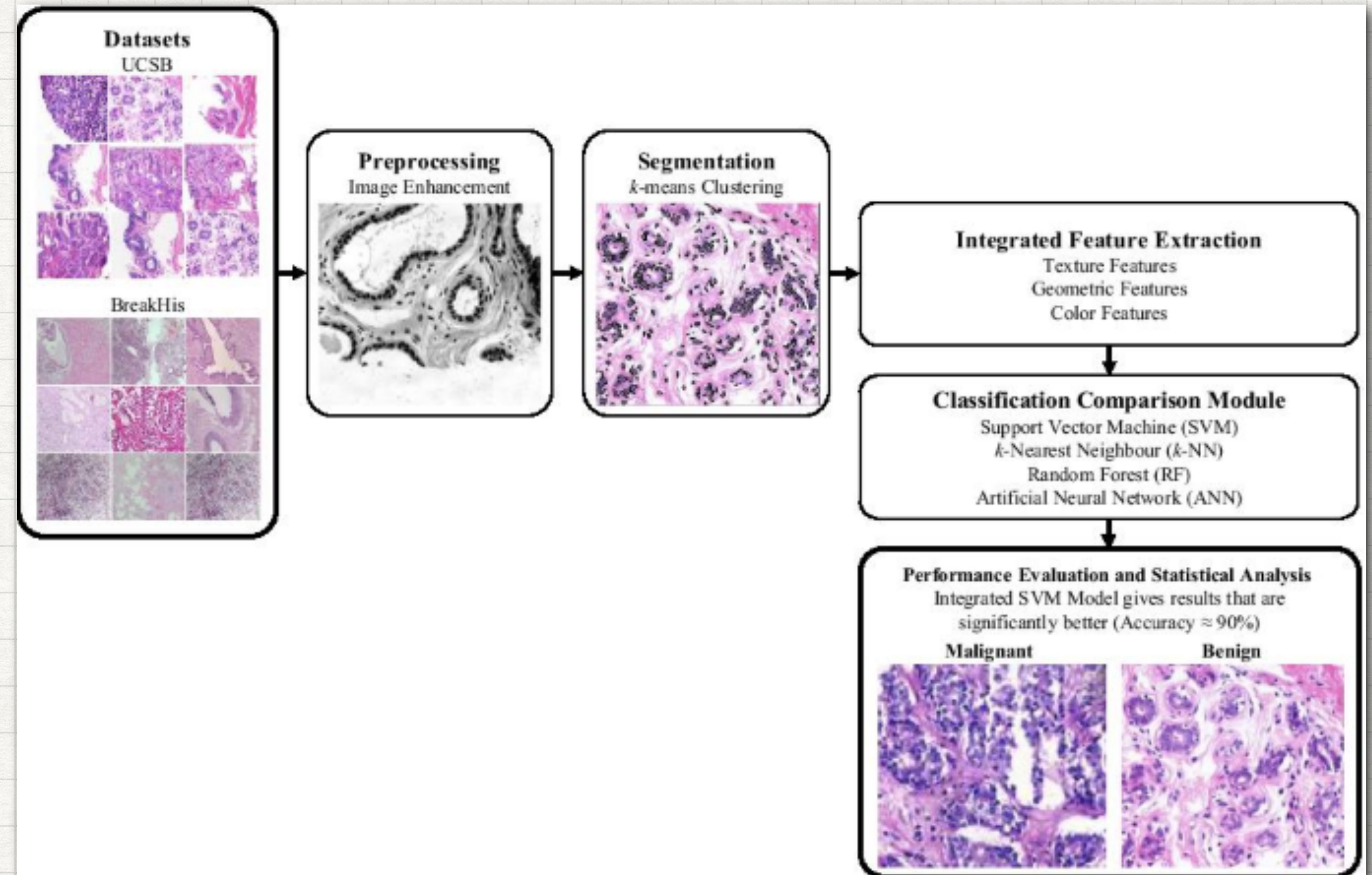


USE-CASES

DISEASE DIAGNOSIS

PATTERN MATCHING

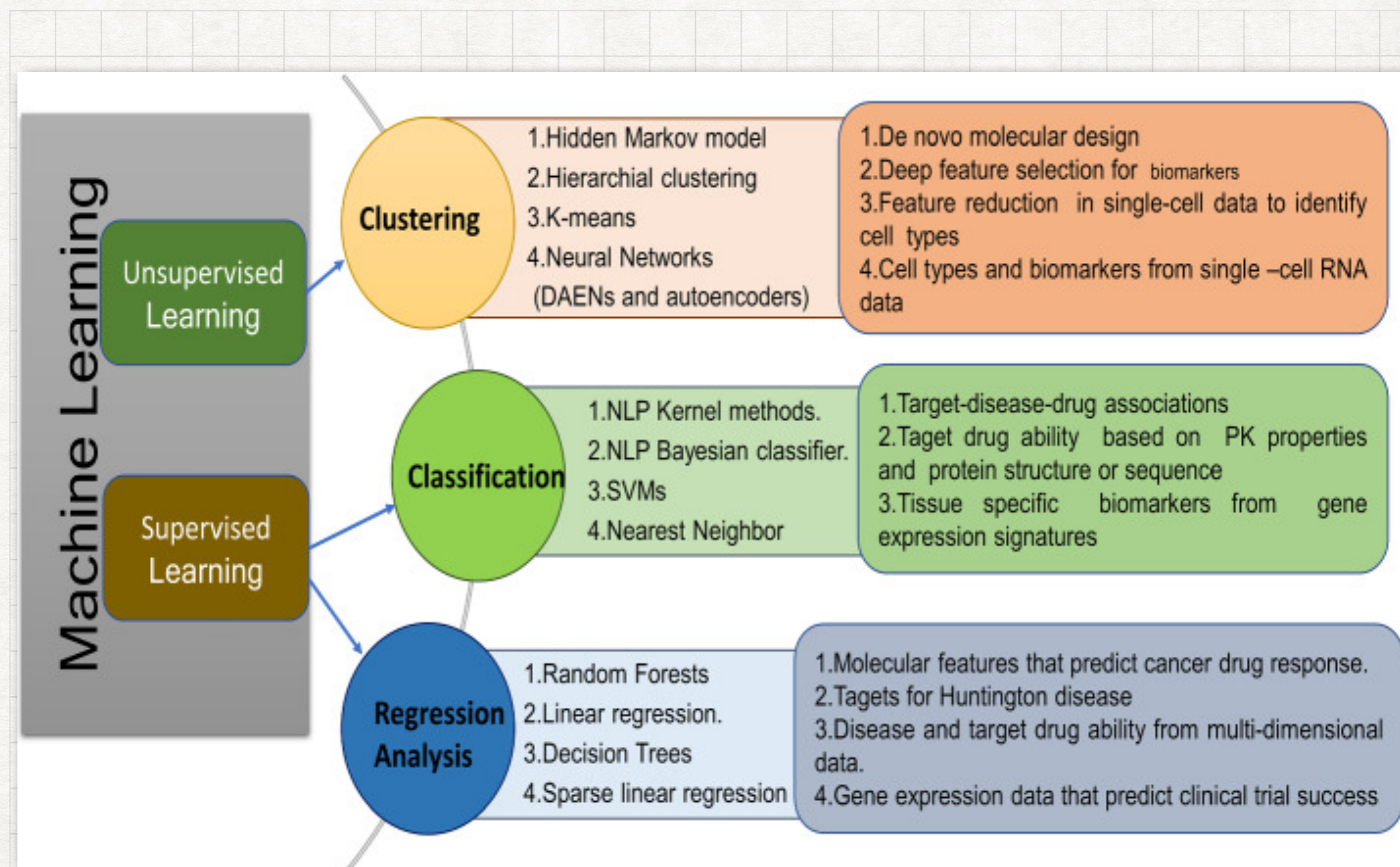
- * ML algorithms, such as support vector machines (SVM) and random forests (RF), are trained on medical imaging data, such as X-rays or MRIs, to identify patterns associated with specific diseases.
- * For example, a SVM model could be trained on a dataset of mammogram images labeled as "normal" or "cancerous" to create a computer-aided detection system for breast cancer.
- * CNN's have exceptional pattern-recognition capabilities.



DRUG DISCOVERY

GRAPHS AND PROPERTIES

- * ML is used to analyze large datasets of chemical compounds and their properties to predict which ones are likely to make effective drugs.
- * For example, a deep learning model called a graph convolutional network (GCN) can be used to learn the structural features of molecules and predict their biological activity.
- * CANDLE project solves large-scale machine learning problems for three cancer-related pilot applications: the drug response problem, RAS pathway problem, and treatment strategy problem (disease-drug) .



PERSONALIZED MEDECINE

LETS GET PERSONAL

- * Machine learning algorithms used to analyze patients' genetic data and other health information to develop personalized treatment plans.
- * For example, a decision tree model could be trained on a dataset of patients with similar characteristics to predict which medication will be most effective for a given patient.
- * Moving towards a Digital Twin...

Digital Twin of a Cancer Patient and Tumor

This example of a digital twin demonstrates the dynamic bidirectional interaction between the real world patient and digital twin to inform clinical decisions regarding interventions including treatments and clinical assessments, which in turn informs the digital twin.

REAL WORLD PATIENT

The patient and the tumor from which data is gathered using various clinical assessments to inform the digital twin.

VVUQ

Verification, validation, and uncertainty quantification

As the patient and tumor are constantly evolving and the data collection can also change over time, VVUQ must occur continually for digital twins.

Uncertainty quantification needs to be addressed for all aspects of the digital twin, including the patient's data, modeling and simulation, and decision making.

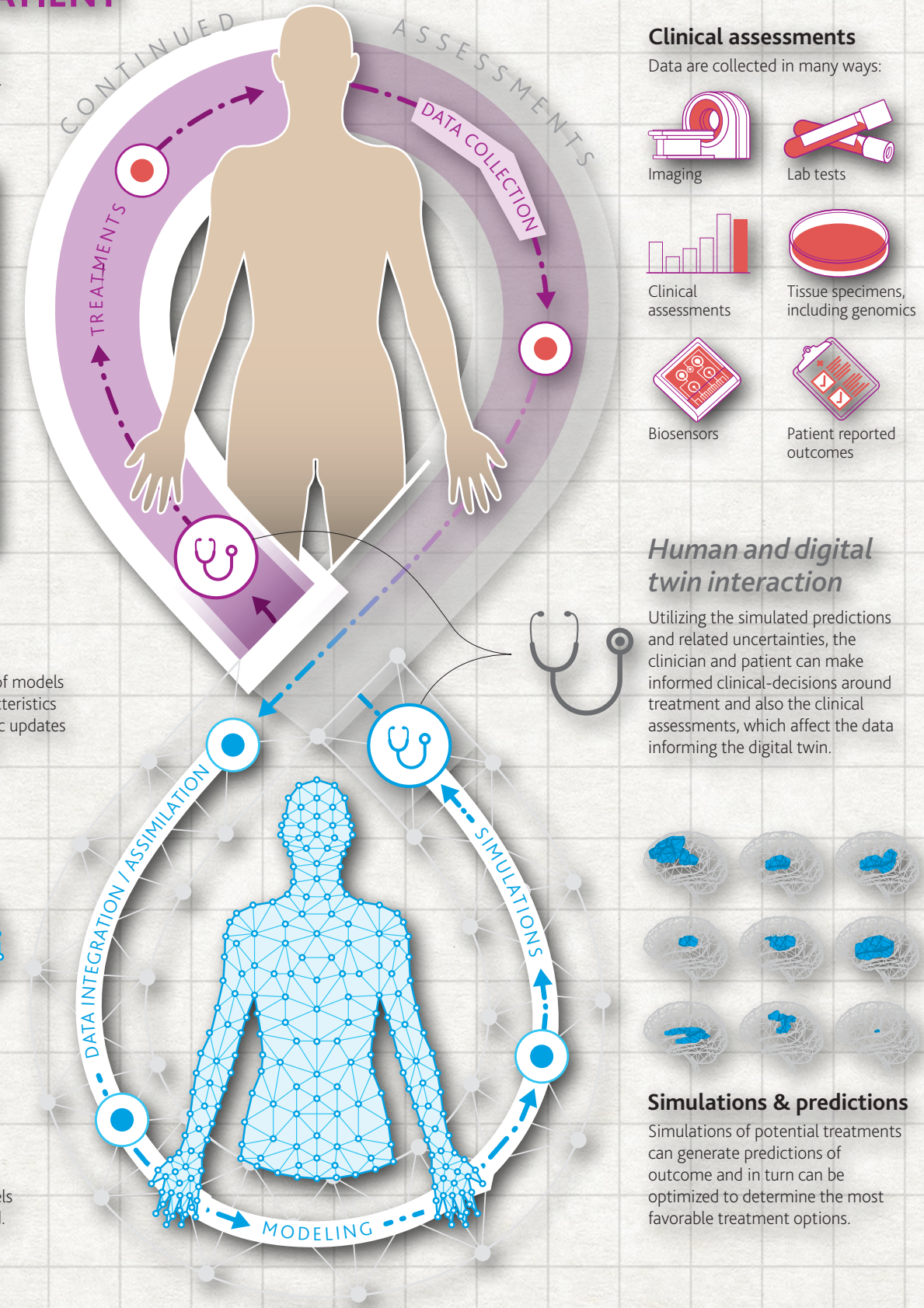
DIGITAL TWIN

The virtual representation comprised of models describing temporal and spatial characteristics of the patient and tumor with dynamic updates using data from the real world patient.

Modeling

Models spanning a range of fidelities and resolutions may be utilized and potentially integrated together.

As new observed data are acquired, the data are assimilated and the models are calibrated, updated, and estimated.



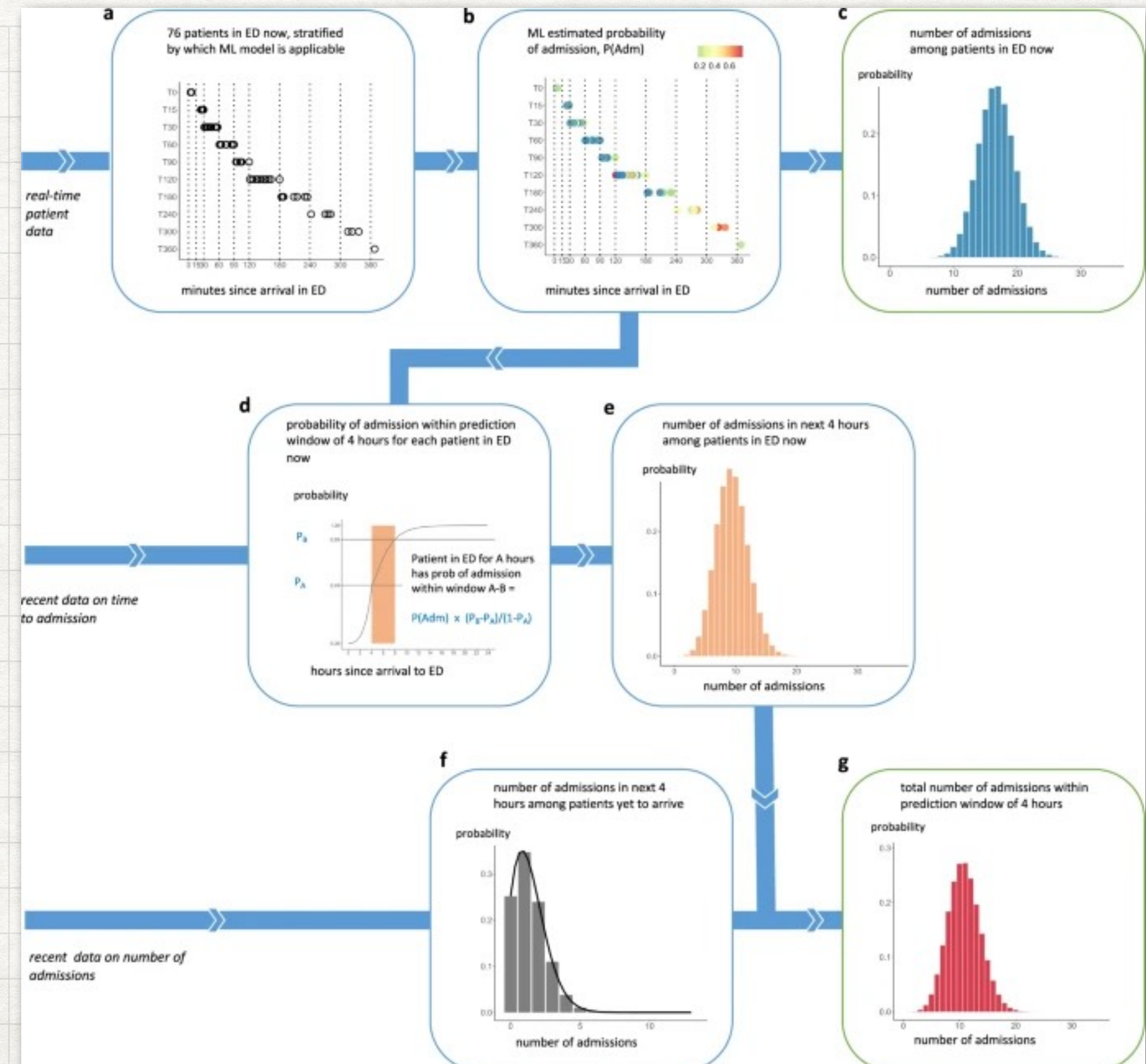
This information is from *Foundational Research Gaps and Future Directions for Digital Twins* available online at www.nationalacademies.org/digital-twins. Copyright by the National Academy of Sciences. All rights reserved.

NATIONAL ACADEMIES
Sciences
Engineering
Medicine

HOSPITAL PLANNING

ADMISSIONS AND READMISSIONS

- * ML models, such as logistic regression and artificial neural networks, can be used to analyze patients' electronic health records (EHR) to identify factors that increase the risk of readmission.
- * This information can then be used to develop interventions to reduce readmissions.
- * Time-series (LSTM) and queuing theory to model admissions fluxes.



Conclusion

SciML for DTs

- Incorporating **scientific knowledge** (almost) always improves the performance of ML algorithms:
- Restricts the space of ML models.
- Stronger inductive bias.
- Can alleviate ML flaws (poor generalisation, optimisation difficulties, lack of interpretability, large amounts of training data).

Conclusion

SciML for DTs

- Incorporating **scientific knowledge** (almost) always improves the performance of ML algorithms:
- Restricts the space of ML models.
- Stronger inductive bias.
- Can alleviate ML flaws (poor generalisation, optimisation difficulties, lack of interpretability, large amounts of training data).
- Incorporating **Machine Learning** in the scientific workflow:
- Enhances the performance (efficiency, accuracy, insights, noise).
- Compensates for unknown/intractable physics.
- Takes advantage of large reservoirs of untapped data.

Thank You!

Contact

- mark.asch@u-picardie.fr
- <https://www.linkedin.com/in/mark-asch-8a257130/>
- <https://github.com/markasch>
- <https://markasch.github.io/DT-tbx-v1/>



REFERENCES

BOOKS & PAPERS

- M. Asch 
- S A Faroughi, N Pawar, C Fernandes, M. Raissi, S. Das, N K Kalantari, S K Mahjour. Physics-Guided, Physics-Informed, and Physics-Encoded Neural Networks in Scientific Computing. arXiv, 2023. <https://arxiv.org/pdf/2211.07377>
- S. Cuomo, V. Schiano Di Cola, F. Giampaolo, G. Rozza, M. Raissi, F. Piccialli. Scientific Machine Learning Through Physics Informed Neural Networks: Where we are and What's Next. Journal of Scientific Computing (2022) 92:88.
- L Lu, P Jin, G Pang, Z Zhang, GE Karniadakis. Learning nonlinear operators via DeepONet based on the universal approximation theorem of operators. Nature Machine Intelligence 3 (3), 218-229 (2021).
- L. Lu, X. Meng, Z. Mao, G. Karniadakis. DeepXDE: A Deep Learning Library for Solving Differential Equations. SIAM Review, 63, 1 (2021).
- E. Darve, K. Xu. Physics constrained learning for data-driven inverse modeling from sparse observations. J. of Computational Physics, 453. (2022).
- M. Raissi, P. Perdikaris, G. E Karniadakis. Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations. J. of Computational Physics, 378, pp. 686-707, 2021.
- N. Kovachki, Z. Li, B. Liu, K. Bhattacharya, A. Stuart, A. Anandkulmar. Neural Operator: Learning Maps Between Function Spaces With Applications to PDEs. J. of Machine Learning Research 24 (2023).
- A. Baydin, B. Pearlmutter, A. Radul, J. Siskind. Automatic differentiation in machine learning: a survey. Journal of Machine Learning Research, 18 (2017), article 153.

